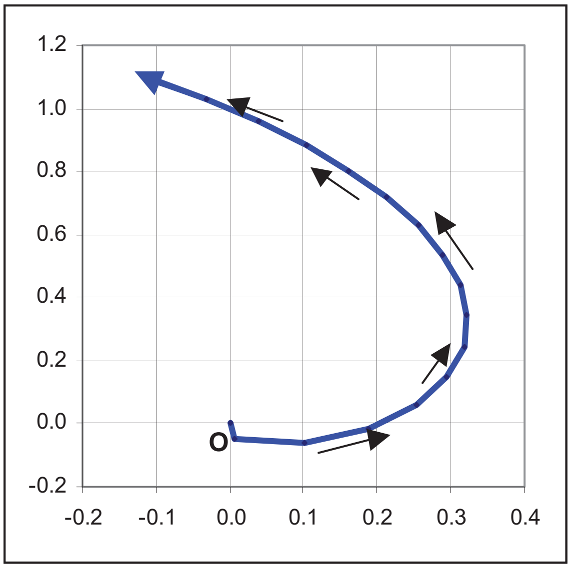

The streamlines are logarithmic spirals moving out from the origin. [They have axisymmetry about O.] This simple distribution is often used to simulate a swirling flow such as a tornado.

4.17 A reasonable approximation for the two-dimensional incompressible laminar boundary layer on the flat surface in Fig. P4.17 is

$$u = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \quad \text{for } y \le \delta$$

where $\delta \approx Cx^{1/2}$, $C = \text{const}$

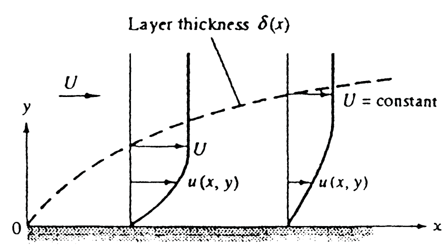

Fig. P4.17

(a) Assuming a no-slip condition at the wall, find an expression for the velocity component $v(x, y)$ for $y \le \delta$. (b) Then find the maximum value of v at the station $x = 1$ m, for the particular case of airflow, when $U = 3$ m/s and $\delta = 1.1$ cm.

Solution: The two-dimensional incompressible continuity equation yields

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -U\left(\frac{-2y}{\delta^2}\frac{d\delta}{dx} + \frac{2y^2}{\delta^3}\frac{d\delta}{dx}\right), \quad \text{or:} \quad v = 2U\frac{d\delta}{dx}\int_0^y \left(\frac{y}{\delta^2} - \frac{y^2}{\delta^3}\right)dy\Big|_{x=const}$$

$$\text{or:} \quad \mathbf{v = 2U\frac{d\delta}{dx}\left(\frac{y^2}{2\delta^2} - \frac{y^3}{3\delta^3}\right)}, \quad \text{where} \quad \frac{d\delta}{dx} = \frac{C}{2\sqrt{x}} = \frac{\delta}{2x} \quad \textit{Ans. (a)}$$

(b) We see that v increases monotonically with y, thus v_{max} occurs at y = δ:

$$v_{max} = v|_{y=\delta} = \frac{\mathbf{U\delta}}{\mathbf{6x}} = \frac{(3\ m/s)(0.011\ m)}{6(1\ m)} = \mathbf{0.0055\ \frac{m}{s}} \quad \textit{Ans.}\text{ (b)}$$

This estimate is within 4% of the exact v_{max} computed from boundary layer theory.

4.18 A piston compresses gas in a cylinder by moving at constant speed V, as in Fig. P4.18. Let the gas density and length at $t = 0$ be ρ_o and L_o, respectively. Let the gas velocity vary linearly from $u = V$ at the piston face to $u = 0$ at $x = L$. If the gas density varies only with time, find an expression for $\rho(t)$.

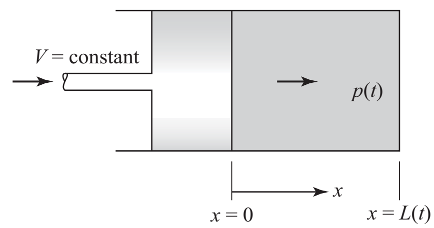

Fig. P4.18

Solution: The one-dimensional unsteady continuity equation reduces to

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = \frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x}, \quad \text{where } u = V\left(1 - \frac{x}{L}\right), \quad L = L_o - Vt, \quad \rho = \rho(t) \text{ only}$$

$$\text{Enter } \frac{\partial u}{\partial x} = -\frac{V}{L} \text{ and separate variables: } \int_{\rho_o}^{\rho}\frac{d\rho}{\rho} = V\int_o^t \frac{dt}{L_o - Vt}$$

$$\text{The solution is } \ln(\rho/\rho_o) = -\ln(1 - Vt/L_o), \quad \text{or: } \rho = \rho_o\left(\frac{\mathbf{L_o}}{\mathbf{L_o - Vt}}\right) \quad \textit{Ans.}$$

4.19 An incompressible flow field has the cylindrical velocity components $\upsilon_\theta = Cr$, $\upsilon_z = K(R^2 - r^2)$, $\upsilon_r = 0$, where C and K are constants and $r \le R$, $z \le L$. Does this flow satisfy continuity? What might it represent physically?

Solution: We check the incompressible continuity relation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial v_z}{\partial z} = 0 = 0 + 0 + 0 \quad \textbf{satisfied identically} \quad \textit{Ans.}$$

This flow also satisfies (cylindrical) momentum and could represent laminar flow inside a tube of radius R whose outer wall (r = R) is rotating at uniform angular velocity.

4.20 A two-dimensional incompressible velocity field has $u = K(1 - e^{-ay})$, for $x \le L$ and $0 \le y \le \infty$. What is the most general form of $v(x, y)$ for which continuity is satisfied and $v = v_o$ at $y = 0$? What are the proper dimensions for constants K and a?