4.23 A tank volume *V* contains gas at conditions (ρ_0, p_0, T_0). At time $t = 0$ it is punctured by a small hole of area *A*. According to the theory of Chap. 9, the mass flow out of such a hole is approximately proportional to *A* and to the tank pressure. If the tank temperature is assumed constant and the gas is ideal, find an expression for the variation of density within the tank.

Solution: This problem is a realistic approximation of the "blowdown" of a high-pressure tank, where the exit mass flow is choked and thus proportional to tank pressure. For a control volume enclosing the tank and cutting through the exit jet, the mass relation is

$$\frac{d}{dt}(m_{tank}) + \dot{m}_{exit} = 0, \quad \text{or:} \quad \frac{d}{dt}(\rho \upsilon) = -\dot{m}_{exit} = -CpA, \quad \text{where } C = \text{constant}$$

$$\text{Introduce} \quad \rho = \frac{p}{RT_o} \quad \text{and separate variables:} \quad \int_{p_o}^{p(t)} \frac{dp}{p} = -\frac{CRT_oA}{\upsilon}\int_o^t dt$$

The solution is an exponential decay of tank density: $\mathbf{p = p_o \exp(-CRT_oAt/\upsilon)}$. *Ans.*

4.24 Reconsider Fig. P4.17 in the following general way. It is known that the boundary layer thickness $\delta(x)$ increases monotonically and that there is no slip at the wall ($y = 0$). Further, $u(x, y)$ merges smoothly with the outer stream flow, where $u \approx U$ = constant outside the layer. Use these facts to prove that (a) the component $\upsilon(x, y)$ is positive everywhere within the layer, (b) υ increases parabolically with *y* very near the wall, and (c) υ is a maximum at $y = \delta$.

Solution: (a) First, if δ is continually increasing with x, then u is continually *decreasing* with x in the boundary layer, that is, $\partial u/\partial x < 0$, hence $\partial v/\partial y = -\partial u/\partial x > \mathbf{0}$ everywhere. It follows that, if $\partial v/\partial y > 0$ and $v = 0$ at $y = 0$, then $\mathbf{v(x, y) > 0}$ **for all** $\mathbf{y \le \delta}$. *Ans.* (a)