(b) At the wall, *u* must be approximately linear with y, if $\tau_w \geq 0$:

$$\text{Near wall: } u \approx y\,f(x), \quad \text{or} \quad \frac{\partial u}{\partial x} = y\frac{df}{dx}, \quad \text{where} \quad \frac{df}{dx} < 0. \quad \text{Then} \quad \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \left(\frac{df}{dx}\right)y$$

$$\text{Thus, near the wall,} \quad v \approx \left(\frac{df}{dx}\right)\int_0^y y\,dy \approx \left(\frac{df}{dx}\right)\frac{y^2}{2} \quad \textbf{Parabolic} \quad \textit{Ans. } \text{(b)}$$

(c) At $y = \delta$, $u \to U$, then $\partial u/\partial x \approx 0$ there and thus $\partial v/\partial y \approx 0$, or $\mathbf{v = v_{max}}$. *Ans.* (c)

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**4.25** An incompressible flow in polar coordinates is given by

$$v_r = K\cos\theta\left(1 - \frac{b}{r^2}\right)$$

$$v_\theta = -K\sin\theta\left(1 + \frac{b}{r^2}\right)$$

Does this field satisfy continuity? For consistency, what should the dimensions of constants *K* and *b* be? Sketch the surface where $v_r = 0$ and interpret.

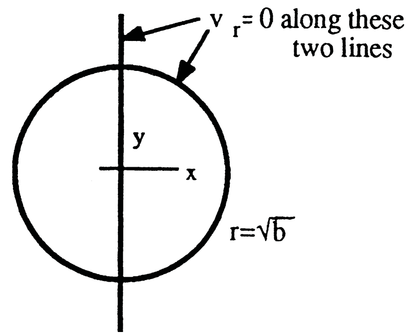


**Fig. P4.25**

**Solution:** Substitute into plane polar coordinate continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0 \stackrel{?}{=} \frac{1}{r}\frac{\partial}{\partial r}\left[K\cos\theta\left(r - \frac{b}{r}\right)\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[-K\sin\theta\left(1 + \frac{b}{r^2}\right)\right] = 0 \quad \textit{Satisfied}$$

The dimensions of *K* <u>must</u> be velocity, {K} = {L/T}, and *b* <u>must</u> be area, $\{b\} = \{L^2\}$. The surfaces where vr     0 are the y-axis and the circle r     (b, as shown above. The pattern represents inviscid flow of a uniform stream past a circular cylinder (Chap. 8).

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**4.26** Curvilinear, or streamline, coordinates are defined in Fig. P4.26, where n is normal to the streamline in the plane of the radius of curvature R. Show that Euler's frictionless momentum equation (4.36) in streamline coordinates becomes

$$-V\frac{\partial \theta}{\partial t} - \frac{V^2}{R} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + g_n \qquad (2)$$

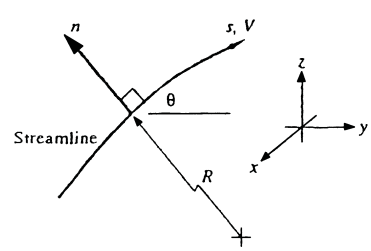


**Fig. P4.26**

Further show that the integral of Eq. (1) with respect to *s* is none other than our old friend Bernoulli's equation (3.76).

**Solution:** This is a laborious derivation, really, **the problem is only meant to *acquaint* the student with streamline coordinates.** The second part is not too hard, though. Multiply the streamwise momentum equation by *ds* and integrate:

$$\frac{\partial V}{\partial t}ds + V\,dV = -\frac{dp}{\rho_2} + g_s\,ds = -\frac{dp}{\rho} - g\sin\theta\,ds = -\frac{dp}{\rho} - g\,dz$$

Integrate from 1 to 2: $\mathbf{\int_1^2 \frac{\partial V}{\partial t}ds + \frac{V_2^2 - V_1^2}{2} + \int_1^2 \frac{dp}{\rho} + g(z_2 - z_1) = 0}$ (Bernoulli) *Ans.*

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**4.27** A frictionless, incompressible steady-flow field is given by

$$\mathbf{V} = 2xy\mathbf{i} - y^2\mathbf{j}$$

in arbitrary units. Let the density be $\rho_0$ = constant and neglect gravity. Find an expression for the pressure gradient in the *x* direction.

**Solution:** For this (gravity-free) velocity, the momentum equation is

$$\rho\left(u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}\right) = -\nabla p, \quad \text{or:} \quad \rho_o[(2xy)(2y\mathbf{i}) + (-y^2)(2x\mathbf{i} - 2y\mathbf{j})] = -\nabla p$$

Solve for $\nabla p = -\rho_o(2xy^2\mathbf{i} + 2y^3\mathbf{j})$, or: $\mathbf{\frac{\partial p}{\partial x} = -\rho_o 2xy^2}$ *Ans.*

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**4.28** If *z* is "up," what are the conditions on constants *a* and *b* for which the velocity field $u = ay$, $\upsilon = bx$, $w = 0$ is an exact solution to the continuity and Navier-Stokes equations for incompressible flow?

**Solution:** First examine the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \stackrel{?}{=} 0 = \frac{\partial}{\partial x}(ay) + \frac{\partial}{\partial y}(bx) + \frac{\partial}{\partial z}(0) = 0 + 0 + 0 \quad \text{for all } a \text{ and } b$$

Given $g_x = g_y = 0$ and w = 0, we need only examine *x*- and *y*-momentum:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \rho[(ay)(0) + (bx)(a)] = -\frac{\partial p}{\partial x} + \mu(0+0)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = \rho[(ay)(b) + (bx)(0)] = -\frac{\partial p}{\partial y} + \mu(0+0)$$