4.35 From the Navier-Stokes equations for incompressible flow in polar coordinates (App. E for cylindrical coordinates), find the most general case of purely circulating motion $\upsilon_\theta(r)$, $\upsilon_r = \upsilon_z = 0$, for flow with no slip between two fixed concentric cylinders, as in Fig. P4.35.

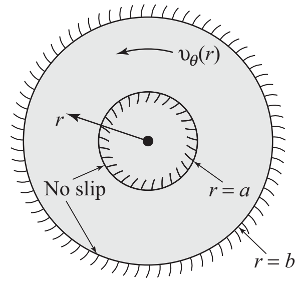

Fig. P4.35

Solution: The preliminary work for this problem is identical to Prob. 4.32 on an earlier page. That is, there are two possible solutions for purely circulating motion υ_θ(r), hence

$$v_\theta = C_1 r + \frac{C_2}{r}, \quad \text{subject to } v_\theta(a) = 0 = C_1 a + C_2/a \quad \text{and} \quad v_\theta(b) = 0 = C_1 b + C_2/b$$

This requires $\mathbf{C_1 = C_2 = 0}$, or $\mathbf{v_\theta = 0}$ (no steady motion possible between fixed walls) *Ans.*

4.36 A constant-thickness film of viscous liquid flows in laminar motion down a plate inclined at angle θ, as in Fig. P4.36. The velocity profile is

$$u = Cy(2h - y) \quad v = w = 0$$

Find the constant C in terms of the specific weight and viscosity and the angle θ. Find the volume flux Q per unit width in terms of these parameters.

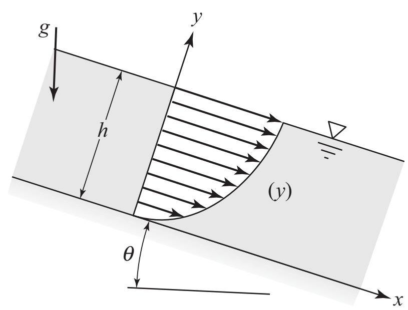

Fig. P4.36

Solution: There is atmospheric pressure all along the surface at y = h, hence $\partial p/\partial x = 0$. The x-momentum equation can easily be evaluated from the known velocity profile:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\nabla^2 u, \quad \text{or:} \quad 0 = 0 + \rho g \sin\theta + \mu(-2C)$$

$$\text{Solve for} \quad \mathbf{C = \frac{\rho g \sin\theta}{2\mu}} \quad \textit{Ans. (a)}$$

The flow rate per unit width is found by integrating the velocity profile and using C:

$$Q = \int_0^h u\,dy = \int_0^h Cy(2h - y)\,dy = \frac{2}{3}Ch^3 = \mathbf{\frac{\rho g h^3 \sin\theta}{3\mu}} \text{ per unit width} \quad \textit{Ans. (b)}$$
