4.41 As mentioned in Sec. 4.10, the velocity profile for laminar flow between two plates, as in Fig. P4.40, is

$$u = \frac{4u_{max}y(h-y)}{h^2} \qquad \upsilon = w = 0$$

If the wall temperature is T_w at both walls, use the incompressible-flow energy equation (4.75) to solve for the temperature distribution $T(y)$ between the walls for steady flow.

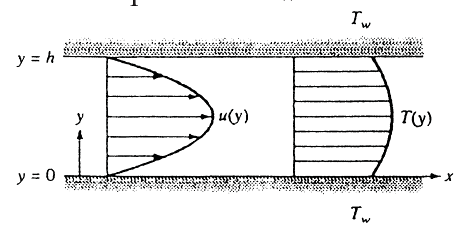

Fig. P4.41

Solution: Assume T = T(y) and use the energy equation with the known u(y):

$$\rho c_p \frac{DT}{dt} = k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2, \quad \text{or:} \quad \rho c_p(0) = k\frac{d^2T}{dy^2} + \mu\left[\frac{4u_{max}}{h^2}(h-2y)\right]^2, \quad \text{or:}$$

$$\frac{d^2T}{dy^2} = -\frac{16\mu u_{max}^2}{kh^4}(h^2 - 4hy + 4y^2), \quad \text{Integrate:} \quad \frac{dT}{dy} = \frac{-16\mu u_{max}^2}{kh^4}\left(h^2y - 2hy^2 + \frac{4y^3}{3} + C_1\right)$$

Before integrating again, note that dT/dy = 0 at y = h/2 (the symmetry condition), so $C_1 = -h^3/6$. Now integrate once more:

$$T = -\frac{16\mu u_{max}^2}{kh^4}\left(h^2\frac{y^2}{2} - 2h\frac{y^3}{3} + \frac{y^4}{3} + C_1 y\right) + C_2$$

If $T = T_w$ at y = 0 and at y = h, then $C_2 = T_w$. The final solution is:

$$\mathbf{T = T_w + \frac{8\mu u_{max}^2}{k}\left[\frac{y}{3h} - \frac{y^2}{h^2} + \frac{4y^3}{3h^3} - \frac{2y^4}{3h^4}\right]} \quad \textit{Ans.}$$

4.42 Suppose that we wish to analyze the rotating, partly-full cylinder of Fig. 2.23 as a *spin-up* problem, starting from rest and continuing until solid-body-rotation is achieved. What are the appropriate boundary and initial conditions for this problem?

Solution: Let $V = V(r, z, t)$. The initial condition is: $V(r, z, 0) = 0$. The boundary conditions are

Along the side walls: $v_\theta(R, z, t) = R\Omega, \quad v_r(R, z, t) = 0, \quad v_z(R, z, t) = 0.$

At the bottom, z = 0: $v_\theta(r, 0, t) = r\Omega, \quad v_r(r, 0, t) = 0, \quad v_z(r, 0, t) = 0.$