4.7 Consider a sphere of radius R immersed in a uniform stream U_0, as shown in Fig. P4.7. According to the theory of Chap. 8, the fluid velocity along streamline AB is given by

$$\mathbf{V} = u\mathbf{i} = U_o\left(1+\frac{R^3}{x^3}\right)\mathbf{i}$$

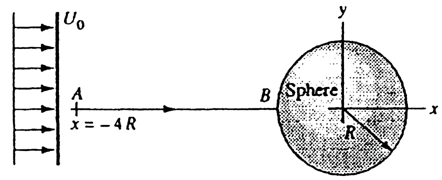

Fig. P4.7

Find (a) the position of maximum fluid acceleration along AB and (b) the time required for a fluid particle to travel from A to B.

Solution: (a) Along this streamline, the fluid acceleration is one-dimensional:

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} = U_o(1+R^3/x^3)(-3U_oR^3/x^4) = -3U_oR^3(x^{-4}+R^3x^{-7}) \quad \text{for } x \le -R$$

The maximum occurs where $d(a_x)/dx = 0$, or at $x = -(7R^3/4)^{1/3} \approx \mathbf{-1.205R}$ *Ans.* (a)
(b) The time required to move along this path from A to B is computed from

$$u = \frac{dx}{dt} = U_o(1+R^3/x^3), \quad \text{or:} \quad \int_{-4R}^{-R}\frac{dx}{1+R^3/x^3} = \int_0^t U_o\,dt,$$

$$\text{or:} \quad U_o t = \left[x - \frac{R}{6}\ln\frac{(x+R)^2}{x^2-Rx+R^2} + \frac{R}{\sqrt{3}}\tan^{-1}\left(\frac{2x-R}{R\sqrt{3}}\right)\right]_{-4R}^{-R} = \infty$$

It takes **an infinite time** to actually *reach* the stagnation point, where the velocity is zero. *Ans.* (b)

4.8 When a valve is opened, fluid flows in the expansion duct of Fig. P4.8 according to the approximation

$$\mathbf{V} = \mathbf{i}U\left(1-\frac{x}{2L}\right)\tanh\frac{Ut}{L}$$

Find (a) the fluid acceleration at $(x, t) = (L, L/U)$ and (b) the time for which the fluid acceleration at $x = L$ is zero. Why does the fluid acceleration become negative after condition (b)?

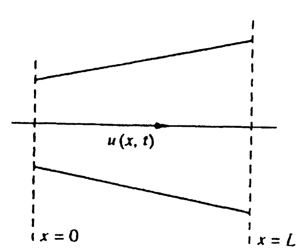

Fig. P4.8