$$\text{At } r = r_o, \quad v_\theta = \Omega_o r_o = C_1 r_o + C_2/r_o$$

Solve for C_1 and C_2. The final result: $\quad \mathbf{v_\theta = \Omega_o r_o \left(\dfrac{r/r_i - r_i/r}{r_o/r_i - r_i/r_o} \right)} \quad$ *Ans.*

This solution <u>may indeed be added to the inner-rotation solution</u>, Eq. (4.146), because the convective acceleration is zero and hence the Navier-Stokes equation is *linear*.

4.82 A solid circular cylinder of radius R rotates at angular velocity Ω in a viscous incompressible fluid which is at rest far from the cylinder, as in Fig. P4.82. Make simplifying assumptions and derive the governing differential equation and boundary conditions for the velocity field v_θ in the fluid. Do not solve unless you are obsessed with this problem. What is the steady-state flow field for this problem?
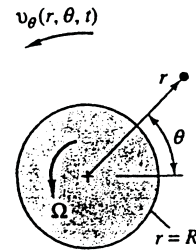

Fig. P4.82

Solution: We assume purely circulating motion: $v_z = v_r = 0$ and $\partial/\partial\theta = 0$. Thus the remaining variables are $v_\theta = \text{fcn}(r, t)$ and $p = \text{fcn}(r, t)$. Continuity is satisfied identically, and the θ-momentum equation reduces to a partial differential equation for v_θ:

$$\frac{\partial v_\theta}{\partial t} = \frac{\mu}{\rho} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_\theta}{\partial r} \right) - \frac{v_\theta}{r^2} \right] \quad \text{subject to } v_\theta(R, t) = \Omega R \quad \text{and} \quad v_\theta(\infty, t) = 0 \quad \textit{Ans.}$$

I am not obsessed with this problem so will not attempt to find a solution. However, at large times, or $t = \infty$, the steady state solution is $\mathbf{v_\theta = \Omega R^2/r}$. *Ans.*

4.83 The flow pattern in bearing lubrication can be illustrated by Fig. P4.83, where a viscous oil (ρ, μ) is forced into the gap $h(x)$ between a fixed slipper block and a wall moving at velocity U. If the gap is thin, $h \ll L$, it can be shown that the pressure and velocity distributions are of the form $p = p(x)$, $u = u(y)$, $v = w = 0$. Neglecting gravity, reduce the Navier-Stokes equations (4.38) to a single differential equation for $u(y)$. What are the proper boundary conditions? Integrate and show that

$$u = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - yh) + U \left(1 - \frac{y}{h} \right)$$

where $h = h(x)$ may be an arbitrary slowly varying gap width. (For further information on lubrication theory, see Ref. 16.)