4.86 SAE 10 oil at 20°C flows between parallel plates 8 mm apart, as in Fig. P4.86. A mercury manometer, with wall pressure taps 1 m apart, registers a 6-cm height, as shown. Estimate the flow rate of oil for this condition.

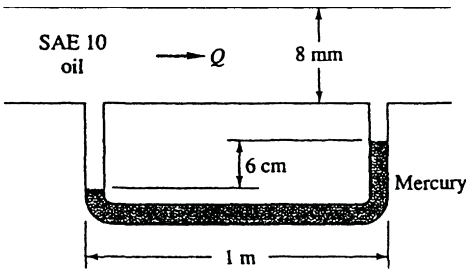

Fig. P4.86

Solution: Assuming laminar flow, this geometry fits Eqs. (4.143, 144) of the text:

$$V_{avg} = \frac{2}{3}u_{max} = \left(\frac{dp}{dx}\right)\frac{h^2}{3\mu}, \quad \text{where } h = \text{plate half-width} = 4\,\text{mm}$$

For SAE 10W oil, take $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. The manometer reads

$$\Delta p = (\rho_{Hg} - \rho_{oil})g\Delta h = (13550 - 870)(9.81)(0.06) \approx 7463 \text{ Pa} \quad \text{for } \Delta x = L = 1\text{m}$$

$$\text{Then} \quad V = \frac{\Delta p}{\Delta x}\frac{h^2}{3\mu} = \left(\frac{7463\text{ Pa}}{1\text{ m}}\right)\frac{(0.004)^2}{3(0.104)} \approx 0.383\ \frac{\text{m}}{\text{s}}$$

$$\text{The flow rate per unit width is} \quad Q = VA = (0.383)(0.008) \approx \mathbf{0.00306}\ \frac{\mathbf{m^3}}{\mathbf{s\cdot m}} \quad \textit{Ans.}$$

4.87 SAE 30W oil at 20°C flows through the 9-cm-diameter pipe in Fig. P4.87 at an average velocity of 4.3 m/s. (a) Verify that the flow is laminar. (b) Determine the volume flow rate in m^3/h. (c) Calculate the expected reading h of the mercury manometer, in cm.

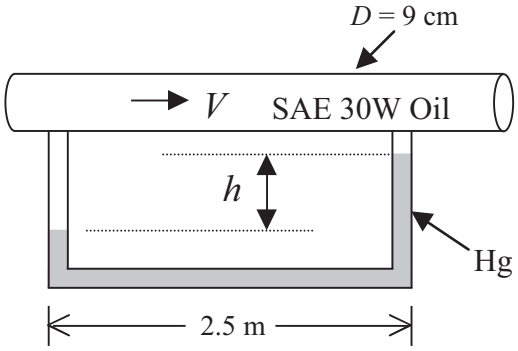

Fig. P4.87

Solution: (a) Check the Reynolds number. For SAE 30W oil, from Appendix A.3, $\rho =$ 891 kg/m^3 and $\mu = 0.29$ kg/(m·s). Then

$$\text{Re}_d = \rho Vd/\mu = (891\text{ kg/m}^3)(4.3\text{ m/s})(0.09\text{ m})/[0.29\text{ kg/(m·s)}] = 1190 < 2000 \textbf{ Laminar} \quad \textit{Ans.}\ \text{(a)}$$

(b) With average velocity known, the volume flow follows easily:

$$Q = AV = [(\pi/4)(0.09\text{ m})^2](4.3\text{ m/s})(3600\text{ s/h}) = \mathbf{98.5\ m^3/h} \quad \textit{Ans.}\ \text{(b)}$$