

resulting velocity distribution. The fluid is at rest

Fig. P4.94

far from the cylinder. [HINT: the cylinder does not induce any radial motion.]

Solution: We already have the useful hint that $v_r = 0$. Continuity then tells us that

$(1/r)\partial v_\theta/\partial\theta = 0$, hence v_θ does not vary with θ . Navier-Stokes then yields the flow. From Eq. D.6, the tangential momentum relation, with $\partial p/\partial\theta = 0$ and $v_\theta = f(r)$, we obtain Eq. (4.139):

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dv_\theta}{dr} \right) = \frac{v_\theta}{r^2}, \quad \text{Solution: } v_\theta = C_1 r + \frac{C_2}{r}$$

As $r \rightarrow \infty$, $v_\theta \rightarrow 0$, hence $C_1 = 0$

$$\text{At } r = R, \quad v_\theta = \Omega R = \frac{C_2}{R}; \quad C_2 = \Omega R^2; \quad \text{Finally, } v_\theta = \frac{\Omega R^2}{r} \quad \text{Ans.}$$

Rotating a cylinder in a large expanse of fluid sets up (eventually) a *potential vortex flow*.

***P4.95** Two immiscible liquids of equal thickness h are being sheared between a fixed and a moving plate, as in Fig. P4.95. Gravity is neglected, and there is no variation with x .

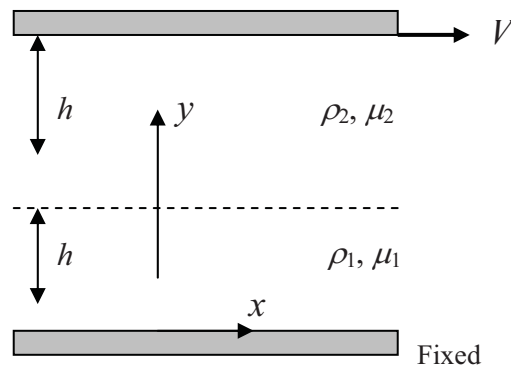


Fig. P4.95

Find an expression for (a) the velocity at the interface; and (b) the shear stress in each fluid. Assume steady laminar flow.