where $\dot{Q}$ = heat flow, J/s
$A$ = surface area, $m^2$
$\Delta T$ = temperature difference, K

The dimensionless form of $h$, called the *Stanton number*, is a combination of $h$, fluid density $\rho$, specific heat $c_p$, and flow velocity $V$. Derive the Stanton number if it is proportional to $h$.

**Solution:** If $\{\dot{Q}\} = \{hA\Delta T\}$, then $\left\{\frac{ML^2}{T^3}\right\} = \{h\}\{L^2\}\{\Theta\}$, or: $\{h\} = \left\{\frac{M}{\Theta T^3}\right\}$

$$\text{Then }\{\text{Stanton No.}\} = \{h^1\rho^b c_p{}^c V^d\} = \left\{\frac{M}{\Theta T^3}\right\}\left\{\frac{M}{L^3}\right\}^b\left\{\frac{L^2}{T^2\Theta}\right\}^c\left\{\frac{L}{T}\right\}^d = M^0L^0T^0\Theta^0$$

Solve for $b = -1,\ c = -1,\text{ and } d = -1.$

$$\text{Thus, finally, Stanton Number} = h\rho^{-1}c_p{}^{-1}V^{-1} = \frac{\mathbf{h}}{\boldsymbol{\rho}\mathbf{Vc_p}} \quad \textit{Ans.}$$

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**5.17** The pressure drop per unit length $\Delta p/L$ in a porous, rotating duct (Really! See Ref. 35) depends upon average velocity $V$, density $\rho$, viscosity $\mu$, duct height $h$, wall injection velocity $v_w$, and rotation rate $\Omega$. Using $(\rho,V,h)$ as repeating variables, rewrite this relationship in dimensionless form.

**Solution:** The relevant dimensions are $\{\Delta p/L\} = \{ML^{-2}T^{-2}\}$, $\{V\} = \{LT^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{h\} = \{L\}$, $\{v_w\} = \{LT^{-1}\}$, and $\{\Omega\} = \{T^{-1}\}$. With $n = 7$ and $j = 3$, we expect $n - j = k = 4$ pi groups: They are found, as specified, using $(\rho, V, h)$ as repeating variables:

$$\Pi_1 = \rho^a V^b h^c \frac{\Delta p}{L} = \left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{M}{L^2T^2}\right\} = M^0L^0T^0, \quad \textit{solve}\ a = -1,\ b = -2,\ c = 1$$

$$\Pi_2 = \rho^a V^b h^c \mu^{-1} = \left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{M}{LT}\right\}^{-1} = M^0L^0T^0, \quad \textit{solve}\ a = 1,\ b = 1,\ c = 1$$

$$\Pi_3 = \rho^a V^b h^c \Omega = \left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{1}{T}\right\} = M^0L^0T^0, \quad \textit{solve}\ a = 0,\ b = -1,\ c = 1$$

$$\Pi_4 = \rho^a V^b h^c v_w = \left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{L}{T}\right\} = M^0L^0T^0, \quad \textit{solve}\ a = 0,\ b = -1,\ c = 0$$

The final dimensionless function then is given by:

$$\Pi_1 = fcn(\Pi_2, \Pi_3, \Pi_4), \quad \text{or:} \quad \frac{\Delta p}{L}\frac{h}{\rho V^2} = fcn\left(\frac{\rho V h}{\mu}, \frac{\Omega h}{V}, \frac{v_w}{V}\right) \quad \textit{Ans.}$$

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**5.18** Under laminar conditions, the volume flow $Q$ through a small triangular-section pore of side length $b$ and length $L$ is a function of viscosity $\mu$, pressure drop per unit length $\Delta p/L$, and $b$. Using the pi theorem, rewrite this relation in dimensionless form. How does the volume flow change if the pore size $b$ is doubled?

**Solution:** Establish the variables and their dimensions:

$$\begin{array}{cccc} Q & = fcn(\Delta p/L, & \mu, & b) \\ \{L^3/T\} & \{M/L^2T^2\} & \{M/LT\} & \{L\} \end{array}$$

Then $n = 4$ and $j = 3$, hence we expect $n - j = 4 - 3 =$ **1** Pi group, found as follows:

$$\Pi_1 = (\Delta p/L)^a (\mu)^b (b)^c Q^1 = \{M/L^2T^2\}^a \{M/LT\}^b \{L\}^c \{L^3/T\}^1 = M^0L^0T^0$$

$$M\text{:}\ a + b = 0; \quad L\text{:}\ -2a - b + c + 3 = 0; \quad T\text{:}\ -2a - b - 1 = 0,$$

$$\text{solve} \quad a = -1,\ b = +1,\ c = -4$$

$$\Pi_1 = \mathbf{\frac{Q\mu}{(\Delta p/L)b^4} = constant} \quad \textit{Ans.}$$

Clearly, if $b$ is doubled, the flow rate $Q$ increases by a factor of **16**. *Ans.*

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**5.19** The period of oscillation $T$ of a water surface wave is assumed to be a function of density $\rho$, wavelength $\lambda$, depth $h$, gravity $g$, and surface tension Y. Rewrite this relationship in dimensionless form. What results if Y is negligible?

**Solution:** Establish the variables and their dimensions:

$$\begin{array}{cccccc} T & = fcn(\rho, & \lambda, & h, & g, & Y) \\ \{T\} & \{M/L^3\} & \{L\} & \{L\} & \{L/T^2\} & \{M/T^2\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 =$ **3** Pi groups, capable of various arrangements and selected by myself as follows:

$$\text{Typical final result:} \quad \mathbf{T(g/\lambda)^{1/2} = fcn\left(\frac{h}{\lambda}, \frac{Y}{\rho g \lambda^2}\right)} \quad \textit{Ans.}$$