If Y is negligible, ρ drops out also, leaving: $\mathbf{T(g/\lambda)^{1/2} = fcn\left(\frac{h}{\lambda}\right)}$ *Ans.*

5.20 We can extend Prob. 5.18 to the case of laminar duct flow of a non-newtonian fluid, for which the simplest relation for stress versus strain-rate is the *power-law* approximation:

$$\tau = C\left(\frac{d\theta}{dt}\right)^n$$

This is the analog of Eq. (1.23). The constant C takes the place of viscosity. If the exponent n is less than (greater than) unity, the material simulates a pseudoplastic (dilatant) fluid, as illustrated in Fig. 1.7. (a) Using the {MLT} system, determine the dimensions of C. (b) The analog of Prob. 5.18 for Power-law laminar triangular-duct flow is $Q = \text{fcn}(C, \Delta p/L, b)$. Rewrite this function in the form of dimensionless Pi groups.

Solution: The shear stress and strain rate have the dimensions $\{\tau\} = \{ML^{-1}T^{-2}\}$, and $\{d\theta/dt\} = \{T^{-1}\}$.

(a) Using these in the equation enables us to find the dimensions of C:

$$\left\{\frac{M}{LT^2}\right\} = \{C\}\left\{\frac{1}{T}\right\}^n, \quad hence\ \mathbf{\{C\} = \left\{\frac{M}{LT^{2-n}}\right\}} \quad Ans.\ \text{(a)}$$

Now that we know {C}, combine it with $\{Q\} = \{L^3T^{-1}\}$, $\{\Delta p/L\} = \{ML^{-2}T^{-2}\}$, and $\{b\} = \{L\}$. Note that there are 4 variables and j = 3 {MLT}, hence we expect 4 − 3 = only ***one*** pi group:

$$\{Q\}^a\left\{\frac{\Delta p}{L}\right\}^b\{L\}^c\{C\} = \left\{\frac{L^3}{T}\right\}^a\left\{\frac{M}{L^2T^2}\right\}^b\{L\}^c\left\{\frac{M}{LT^{2-n}}\right\} = M^0L^0T^0,$$

$$solve \quad a = n,\ b = -1,\ c = -3n - 1$$

The one and only dimensionless pi group is thus:

$$\mathbf{\Pi_1 = \frac{Q^nC}{(\Delta p/L)b^{3n+1}} = constant} \quad Ans.\ \text{(b)}$$

5.21 In Example 5.1 we used the pi theorem to develop Eq. (5.2) from Eq. (5.1). Instead of merely listing the primary dimensions of each variable, some workers list the *powers* of each primary dimension for each variable in an array:

$$\begin{array}{c} \\ M \\ L \\ T \end{array}\begin{array}{c}\begin{array}{ccccc} F & L & U & \rho & \mu \end{array}\\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & -3 & -1 \\ -2 & 0 & -1 & 0 & -1 \end{bmatrix}\end{array}$$

This array of exponents is called the *dimensional matrix* for the given function. Show that the *rank* of this matrix (the size of the largest nonzero determinant) is equal to $j = n - k$, the desired reduction between original variables and the pi groups. This is a general property of dimensional matrices, as noted by Buckingham [29].

Solution: The **rank** of a matrix is the size of the largest submatrix within which has a *non-zero determinant*. This means that the constants in that submatrix, when considered as coefficients of algebraic equations, are *linearly independent*. Thus we establish the number of *independent* parameters—adding one more forms a dimensionless group. For the example shown, the rank is **three** (note the very first 3 × 3 determinant on the left has a non-zero determinant). Thus "j" = 3 for the drag force system of variables.

5.22 The angular velocity Ω of a windmill is a function of windmill diameter D, wind velocity V, air density ρ, windmill height H as compared to atmospheric boundary layer height L, and the number of blades N: $\Omega = \text{fcn}(D, V, \rho, H/L, N)$. Viscosity effects are negligible. Rewrite this function in terms of dimensionless Pi groups.

Solution: We have $n = 6$ variables, $j = 3$ dimensions (M, L, T), thus expect $n - j = 3$ Pi groups. Since only ρ has *mass* dimensions, it drops out. After some thought, we realize that H/L and N are already dimensionless! The desired dimensionless function becomes:

$$\frac{\mathbf{\Omega D}}{\mathbf{V}} = \boldsymbol{fcn}\left(\frac{\mathbf{H}}{\mathbf{L}}, \mathbf{N}\right) \quad \textit{Ans.}$$

5.23 The period T of vibration of a beam is a function of its length L, area moment of inertia I, modulus of elasticity E, density ρ, and Poisson's ratio σ. Rewrite this relation in dimensionless form. What further reduction can we make if E and I can occur only in the product form EI?

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} \text{T} = \text{fcn}(& \text{L}, & \text{I}, & \text{E}, & \rho, & \sigma\ ) \\ \{\text{T}\} & \{\text{L}\} & \{\text{L}^4\} & \{\text{M/LT}^2\} & \{\text{M/L}^3\} & \{\text{none}\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 = \mathbf{3}$ Pi groups, capable of various arrangements and selected by myself as follows: [Note that σ must be a Pi group.]