(b) If we swing the pendulum on the moon at the same 20°, we may use similarity:

$$T_1\left(\frac{g_1}{L_1}\right)^{1/2} = (2.04\text{ s})\left(\frac{9.81\text{ m/s}^2}{1.0\text{ m}}\right)^{1/2} = 6.39 = T_2\left(\frac{1.62\text{ m/s}^2}{0.3\text{ m}}\right)^{1/2},$$

$$\text{or:}\quad \mathbf{T_2 = 2.75\ s}\quad \textit{Ans.}\text{(b)}$$

5.27 In studying sand transport by ocean waves, A. Shields in 1936 postulated that the bottom shear stress τ required to move particles depends upon gravity g, particle size d and density ρ_p, and water density ρ and viscosity μ. Rewrite this in terms of dimensionless groups (which led to the *Shields Diagram* in 1936).

Solution: There are six variables (τ, g, d, ρ_p, ρ, μ) and three dimensions (M, L, T), hence we expect $n - j = 6 - 3 = \mathbf{3}$ Pi groups. The author used (ρ, g, d) as repeating variables:

$$\boldsymbol{\frac{\tau}{\rho g d} = fcn\left(\frac{\rho g^{1/2} d^{3/2}}{\mu}, \frac{\rho_p}{\rho}\right)}\quad \textit{Ans.}$$

The shear parameter used by Shields himself was based on *net* weight: $\boldsymbol{\tau/[(\rho_p - \rho)gd]}$.

5.28 A simply supported beam of diameter D, length L, and modulus of elasticity E is subjected to a fluid crossflow of velocity V, density ρ, and viscosity μ. Its center deflection δ is assumed to be a function of all these variables. (a) Rewrite this proposed function in dimensionless form. (b) Suppose it is known that δ is independent of μ, inversely proportional to E, and dependent only upon ρV^2, not ρ and V separately. Simplify the dimensionless function accordingly.

Solution: Establish the variables and their dimensions:

$$\underset{\{L\}}{\delta} = \text{fcn}(\underset{\{M/L^3\}}{\rho}, \underset{\{L\}}{D}, \underset{\{L\}}{L}, \underset{\{M/LT^2\}}{E}, \underset{\{L/T\}}{V}, \underset{\{M/LT\}}{\mu})$$

Then $n = 7$ and $j = 3$, hence we expect $n - j = 7 - 3 = \mathbf{4}$ Pi groups, capable of various arrangements and selected by myself, as follows (a):

$$\text{Well-posed final result:}\quad \boldsymbol{\frac{\delta}{L} = fcn\left(\frac{L}{D}, \frac{\rho V D}{\mu}, \frac{E}{\rho V^2}\right)}\quad \textit{Ans.}\text{ (a)}$$

(b) If μ is unimportant, then the Reynolds number ($\rho VD/\mu$) drops out, and we have already cleverly combined E with ρV^2, which we can now slip out upside down:

$$\text{If } \mu \text{ drops out and } \quad \delta \propto \frac{1}{E}, \quad \text{then } \frac{\delta}{L} = \frac{\rho V^2}{E}\,\text{fcn}\left(\frac{L}{D}\right),$$

$$\text{or:} \quad \frac{\boldsymbol{\delta E}}{\boldsymbol{\rho V^2 L}} = \textbf{fcn}\left(\frac{\mathbf{L}}{\mathbf{D}}\right) \quad \textit{Ans.}\text{ (b)}$$

5.29 When fluid in a pipe is accelerated linearly from rest, it begins as laminar flow and then undergoes transition to turbulence at a time t_{tr} which depends upon the pipe diameter D, fluid acceleration a, density ρ, and viscosity μ. Arrange this into a dimensionless relation between t_{tr} and D.

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} t_{tr} & = \text{fcn}(& \rho & , \; D \; , & a & , \quad \mu \quad) \\ \{T\} & & \{M/L^3\} & \{L\} & \{L/T^2\} & \{M/LT\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect $n - j = 5 - 3 = \mathbf{2}$ Pi groups, capable of various arrangements and selected by myself, as required, to isolate t_{tr} versus D:

$$\mathbf{t_{tr}}\left(\frac{\boldsymbol{\rho}\mathbf{a^2}}{\boldsymbol{\mu}}\right)^{\mathbf{1/3}} = \textbf{fcn}\left[\mathbf{D}\left(\frac{\boldsymbol{\rho}^2\mathbf{a}}{\boldsymbol{\mu}^2}\right)^{\mathbf{1/3}}\right] \quad \textit{Ans.}$$

5.30 The wall shear stress τ_w for flow in a narrow annular gap between a fixed and a rotating cylinder is a function of density ρ, viscosity μ, angular velocity Ω, outer radius R, and gap width Δr. Using (ρ, Ω, R) as repeating variables, rewrite this relation in dimensionless form.

Solution: The relevant dimensions are $\{\tau_w\} = \{ML^{-1}T^{-2}\}$, $\{\rho\} = \{ML^{-3}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{\Omega\} = \{T^{-1}\}$, $\{R\} = \{L\}$, and $\{\Delta r\} = \{L\}$. With $n = 6$ and $j = 3$, we expect $n - j = k = 3$ pi groups. They are found, as specified, using (ρ, Ω, R) as repeating variables:

$$\Pi_1 = \rho^a \Omega^b R^c \tau_w = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{1}{T}\right\}^b \{L\}^c \left\{\frac{M}{LT^2}\right\} = M^0 L^0 T^0, \quad \textit{solve} \;\; a = -1, b = -2, c = -2$$

$$\Pi_2 = \rho^a \Omega^b R^c \mu^{-1} = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{1}{T}\right\}^b \{L\}^c \left\{\frac{M}{LT}\right\}^{-1} = M^0 L^0 T^0, \quad \textit{solve} \;\; a = 1, b = 1, c = 2$$

$$\Pi = \rho^a \Omega^b R^c \Delta r = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{1}{T}\right\}^b \{L\}^c \{L\} = M^0 L^0 T^0, \quad \textit{solve} \;\; a = 0, b = 0, c = -1$$

The final dimensionless function has the form:

$$\Pi_1 = fcn(\Pi_2, \Pi_3), \quad or: \quad \frac{\boldsymbol{\tau_{wall}}}{\boldsymbol{\rho\Omega^2 R^2}} = \boldsymbol{fcn}\left(\frac{\boldsymbol{\rho\Omega R^2}}{\boldsymbol{\mu}}, \frac{\boldsymbol{\Delta r}}{\boldsymbol{R}}\right) \quad \textit{Ans.}$$