Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} M & = \text{fcn}(& R, & \Omega, & \mu, & \theta\ ) \\ \{ML^2/T^2\} & & \{L\} & \{1/T\} & \{M/LT\} & \{1\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect $n - j = 5 - 3 = \mathbf{2}$ Pi groups, capable of only one reasonable arrangement, as follows:

$$\frac{\mathbf{M}}{\mu\Omega\mathbf{R}^3} = \mathbf{fcn}(\boldsymbol{\theta}); \quad \text{if } M \propto \theta, \quad \text{then } \frac{\mathbf{M}}{\mu\Omega\theta\mathbf{R}^3} = \mathbf{constant} \quad \textit{Ans.}$$

See Prob. 1.56 of this Manual, for an analytical solution.

5.36 The rate of heat loss, Q_{loss} through a window is a function of the temperature difference ΔT, the surface area A, and the R resistance value of the window (in units of $ft^2\cdot hr\cdot °F/Btu$): Q_{loss} = fcn(ΔT, A, R). (a) Rewrite in dimensionless form. (b) If the temperature difference doubles, how does the heat loss change?

Solution: First figure out the dimensions of R: $\{R\} = \{T^3\Theta/M\}$. Then note that $n = 4$ variables and $j = 3$ dimensions, hence we expect only 4 − 3 = *one* Pi group, and it is:

$$\Pi_1 = \frac{Q_{loss}R}{A\,\Delta T} = Const, \quad or: \quad \boldsymbol{Q_{loss} = Const\,\frac{A\,\Delta T}{R}} \quad \textit{Ans. (a)}$$

(b) Clearly (to me), Q ∝ ΔT: **if Δt doubles, Q_{loss} also doubles**. *Ans.* (b)

P5.37 The volume flow Q through an orifice plate is a function of pipe diameter D, pressure drop Δp across the orifice, fluid density ρ and viscosity μ, and orifice diameter d. Using D, ρ, and Δp as repeating variables, express this relationship in dimensionless form.

Solution: There are 6 variables and 3 primary dimensions (MLT), and we already know that

$j = 3$, because the problem thoughtfully gave the repeating variables. Use the pi theorem to find the three pi's:

$$\Pi_1 = D^a\rho^b\Delta p^c Q \ ; \ \text{Solve for } a = -2,\ b = 1/2,\ c = -1/2. \quad \text{Thus} \quad \Pi_1 = \frac{Q\rho^{1/2}}{D^2\,\Delta p^{1/2}}$$

$$\Pi_2 = D^a\rho^b\Delta p^c d \ ; \ \text{Solve for } a = -1\ \ b = 0\ \ c = 0. \quad \text{Thus} \quad \Pi_1 = \frac{d}{D}$$

$$\Pi_3 = D^a\rho^b\Delta p^c \mu \ ; \ \text{Solve for } a = -1,\ b = -1/2,\ c = -1/2. \quad \text{Thus} \quad \Pi_1 = \frac{\mu}{D\rho^{1/2}\Delta p^{1/2}}$$