Solution: (a) Since all terms in the equation contain C, we establish the dimensions of k and $\mathcal{D}$ by comparing $\{k\}$ and $\{\mathcal{D}\partial^2/\partial x^2\}$ to $\{u\partial/\partial x\}$:

$$\{k\} = \{\mathcal{D}\}\left\{\frac{\partial^2}{\partial x^2}\right\} = \{\mathcal{D}\}\left\{\frac{1}{L^2}\right\} = \{u\}\left\{\frac{\partial}{\partial x}\right\} = \left\{\frac{L}{T}\right\}\left\{\frac{1}{L}\right\},$$

$$\text{hence} \quad \boldsymbol{\{k\} = \left\{\frac{1}{T}\right\}} \quad \text{and} \quad \boldsymbol{\{\mathcal{D}\} = \left\{\frac{L^2}{T}\right\}} \quad \textit{Ans. (a)}$$

(b) To non-dimensionalize the equation, define $u^* = u/V$, $t^* = Vt/L$, and $x^* = x/L$ and sub-stitute into the basic partial differential equation. The dimensionless result is

$$\boldsymbol{u^*\frac{\partial C}{\partial x^*} = \left(\frac{\mathcal{D}}{VL}\right)\frac{\partial^2 C}{\partial x^{*2}} - \left(\frac{kL}{V}\right)C - \frac{\partial C}{\partial t^*}}, \text{ where } \frac{VL}{\mathcal{D}} = \text{mass-transfer Peclet number} \quad \textit{Ans. (b)}$$

5.46 The differential equation for compressible inviscid flow of a gas in the xy plane is

$$\frac{\partial^2\phi}{\partial t^2} + \frac{\partial}{\partial t}(u^2+\upsilon^2) + (u^2-a^2)\frac{\partial^2\phi}{\partial x^2} + (\upsilon^2-a^2)\frac{\partial^2\phi}{\partial y^2} + 2u\upsilon\frac{\partial^2\phi}{\partial x\,\partial y} = 0$$

where ϕ is the velocity potential and a is the (variable) speed of sound of the gas. Nondimensionalize this relation, using a reference length L and the inlet speed of sound a_o as parameters for defining dimensionless variables.

Solution: The appropriate dimensionless variables are $u^* = u/a_o$, $t^* = a_o t/L$, $x^* = x/L$, $a^* = a/a_o$, and $\phi^* = \phi/(a_o L)$. Substitution into the PDE for ϕ as above yields

$$\boldsymbol{\frac{\partial^2\phi^*}{\partial t^{*2}} + \frac{\partial}{\partial t^*}(u^{*2}+v^{*2}) + (u^{*2}-a^{*2})\frac{\partial^2\phi^*}{\partial x^{*2}} + (v^{*2}-a^{*2})\frac{\partial^2\phi^*}{\partial y^{*2}} + 2u^*v^*\frac{\partial^2\phi^*}{\partial x^*\partial y^*} = 0} \quad \textit{Ans.}$$

The PDE comes clean and there are no dimensionless parameters. *Ans.*

5.47 The differential equation for small-amplitude vibrations $y(x, t)$ of a simple beam is given by

$$\rho A\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} = 0$$

where ρ = beam material density
A = cross-sectional area
I = area moment of inertia
E = Young's modulus