Solution: (a) Since all terms in the equation contain C, we establish the dimensions of *k* and $\mathcal{D}$ by comparing $\{k\}$ and $\{\mathcal{D}\partial^2/\partial x^2\}$ to $\{u\partial/\partial x\}$:

$$\{k\} = \{\mathcal{D}\}\left\{\frac{\partial^2}{\partial x^2}\right\} = \{\mathcal{D}\}\left\{\frac{1}{L^2}\right\} = \{u\}\left\{\frac{\partial}{\partial x}\right\} = \left\{\frac{L}{T}\right\}\left\{\frac{1}{L}\right\},$$

$$\text{hence} \quad \mathbf{\{k\} = \left\{\frac{1}{T}\right\}} \quad \text{and} \quad \mathbf{\{\mathcal{D}\} = \left\{\frac{L^2}{T}\right\}} \quad \textit{Ans. (a)}$$

(b) To non-dimensionalize the equation, define $u^* = u/V$, $t^* = Vt/L$, and $x^* = x/L$ and sub-stitute into the basic partial differential equation. The dimensionless result is

$$\boldsymbol{u^*\frac{\partial C}{\partial x^*} = \left(\frac{\mathcal{D}}{VL}\right)\frac{\partial^2 C}{\partial x^{*2}} - \left(\frac{kL}{V}\right)C - \frac{\partial C}{\partial t^*}}, \text{ where } \frac{VL}{\mathcal{D}} = \text{mass-transfer Peclet number} \quad \textit{Ans. (b)}$$

5.46 The differential equation for compressible inviscid flow of a gas in the *xy* plane is

$$\frac{\partial^2\phi}{\partial t^2} + \frac{\partial}{\partial t}(u^2 + \upsilon^2) + (u^2 - a^2)\frac{\partial^2\phi}{\partial x^2} + (\upsilon^2 - a^2)\frac{\partial^2\phi}{\partial y^2} + 2u\upsilon\frac{\partial^2\phi}{\partial x\,\partial y} = 0$$

where ϕ is the velocity potential and *a* is the (variable) speed of sound of the gas. Nondimensionalize this relation, using a reference length *L* and the inlet speed of sound a_o as parameters for defining dimensionless variables.

Solution: The appropriate dimensionless variables are $u^* = u/a_o$, $t^* = a_o t/L$, $x^* = x/L$, $a^* = a/a_o$, and $\phi^* = \phi/(a_o L)$. Substitution into the PDE for ϕ as above yields

$$\boldsymbol{\frac{\partial^2\phi^*}{\partial t^{*2}} + \frac{\partial}{\partial t^*}(u^{*2} + v^{*2}) + (u^{*2} - a^{*2})\frac{\partial^2\phi^*}{\partial x^{*2}} + (v^{*2} - a^{*2})\frac{\partial^2\phi^*}{\partial y^{*2}} + 2u^*v^*\frac{\partial^2\phi^*}{\partial x^*\partial y^*} = 0} \quad \textit{Ans.}$$

The PDE comes clean and there are no dimensionless parameters. *Ans.*

5.47 The differential equation for small-amplitude vibrations *y*(*x*, *t*) of a simple beam is given by

$$\rho A\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} = 0$$

where ρ = beam material density
A = cross-sectional area
I = area moment of inertia
E = Young's modulus

Use only the quantities ρ, E, and A to nondimensionalize y, x, and t, and rewrite the differential equation in dimensionless form. Do any parameters remain? Could they be removed by further manipulation of the variables?

Solution: The appropriate dimensionless variables are

$$y^* = \frac{y}{\sqrt{A}}; \quad t^* = t\sqrt{\frac{E}{\rho A}}; \quad x^* = \frac{x}{\sqrt{A}}$$

Substitution into the PDE above yields a dimensionless equation with *one* parameter:

$$\frac{\partial^2 y^*}{\partial t^{*2}} + \left(\frac{I}{A^2}\right)\frac{\partial^4 y^*}{\partial x^{*4}} = 0; \quad \text{One geometric parameter: } \frac{I}{A^2} \quad \textit{Ans.}$$

We could *remove* (I/A^2) completely by redefining $\mathbf{x^* = x/I^{1/4}}$. *Ans.*

5.48 A smooth steel (SG = 7.86) sphere is immersed in a stream of ethanol at 20°C moving at 1.5 m/s. Estimate its drag in N from Fig. 5.3*a*. What stream velocity would quadruple its drag? Take D = 2.5 cm.

Solution: For ethanol at 20°C, take $\rho \approx 789$ kg/m^3 and $\mu \approx 0.0012$ kg/m·s. Then

$$Re_D = \frac{\rho UD}{\mu} = \frac{789(1.5)(0.025)}{0.0012} \approx 24700; \quad \text{Read Fig. 5.3(a):} \quad C_{D,sphere} \approx 0.4$$

$$\text{Compute drag} \quad F = C_D\left(\frac{1}{2}\right)\rho U^2 \frac{\pi}{4}D^2 = (0.4)\left(\frac{1}{2}\right)(789)(1.5)^2\left(\frac{\pi}{4}\right)(0.025)^2$$

$$\approx \mathbf{0.17\ N} \quad \textit{Ans.}$$

Since $C_D \approx$ constant in this range of Re_D, **doubling U quadruples** the drag. *Ans.*

5.49 The sphere in Prob. 5.48 is dropped in gasoline at 20°C. Ignoring its acceleration phase, what will its terminal (constant) fall velocity be, from Fig. 5.3*a*?

Solution: For gasoline at 20°C, take $\rho \approx 680$ kg/m^3 and $\mu \approx 2.92\text{E}-4$ kg/m·s. For steel take $\rho \approx 7800$ kg/m^3. Then, in "terminal" velocity, the net weight equals the drag force:

$$\text{Net weight} = (\rho_{steel} - \rho_{gasoline})g\frac{\pi}{6}D^3 = \text{Drag force,}$$

$$\text{or:} \quad (7800 - 680)(9.81)\frac{\pi}{6}(0.025)^3 = 0.571\ N = C_D\left(\frac{1}{2}\right)(680)U^2\frac{\pi}{4}(0.025)^2$$