Solution: Given $\Delta p = fcn(\rho, V, d/D)$, then by dimensional analysis $\Delta p/(\rho V^2) = fcn(d/D)$. For water at 20°C, take $\rho = 998\ kg/m^3$. For gasoline at 20°C, take $\rho = 680\ kg/m^3$. Then, using the water 'model' data to obtain the function "fcn(d/D)", we calculate

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{5000}{(998)(4.0)^2} = 0.313 = \frac{\Delta p_p}{\rho_p V_p^2} = \frac{15000}{(680)V_p^2}, \quad \text{solve for } V_p \approx 8.39\ \frac{m}{s}$$

$$\text{Given} \quad Q = \frac{9}{60}\ \frac{m^3}{s} = V_p A_p = (8.39)\frac{\pi}{4}D_p^2, \quad \text{solve for best } \mathbf{D_p \approx 0.151\ m} \quad \textit{Ans.}$$

5.72 A one-fifteenth-scale model of a parachute has a drag of 450 lbf when tested at 20 ft/s in a water tunnel. If Reynolds-number effects are negligible, estimate the terminal fall velocity at 5000-ft standard altitude of a parachutist using the prototype if chute and chutist together weigh 160 lbf. Neglect the drag coefficient of the woman.

Solution: For water at 20°C, take $\rho = 1.94\ kg/m^3$. For air at 5000-ft standard altitude (Table A-6) take $\rho = 0.00205\ kg/m^3$. If Reynolds number is unimportant, then the two cases have the same drag-force coefficient:

$$C_{Dm} = \frac{F_m}{\rho_m V_m^2 D_m^2} = \frac{450}{1.94(20)^2(D_p/15)^2} = C_{Dp} = \frac{160}{0.00205 V_p^2 D_p^2},$$

$$\text{solve} \quad \mathbf{V_p \approx 24.5\ \frac{ft}{s}} \quad \textit{Ans.}$$

5.73 The power *P* generated by a certain windmill design depends upon its diameter *D*, the air density ρ, the wind velocity *V*, the rotation rate Ω, and the number of blades *n*. (a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when *V* = 40 m/s and when rotating at 4800 rev/min. (b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude? (c) What is the appropriate rotation rate of the prototype?

Solution: (a) For the function $P = fcn(D, \rho, V, \Omega, n)$ the appropriate dimensions are $\{P\} = \{ML^2T^{-3}\}$, $\{D\} = \{L\}$, $\{\rho\} = \{ML^{-3}\}$, $\{V\} = \{L/T\}$, $\{\Omega\} = \{T^{-1}\}$, and $\{n\} = \{1\}$. Using (D, ρ, V) as repeating variables, we obtain the desired dimensionless function:

$$\boldsymbol{\frac{P}{\rho D^2 V^3} = fcn\left(\frac{\Omega D}{V}, n\right)} \quad \textit{Ans.}\ \text{(a)}$$