6.27 Let us attack Prob. 6.25 in symbolic fashion, using Fig. P6.27. All parameters are constant except the upper tank depth *Z*(*t*). Find an expression for the flow rate *Q*(*t*) as a function of *Z*(*t*). Set up a differential equation, and solve for the time t_0 to drain the upper tank completely. Assume quasi-steady laminar flow.

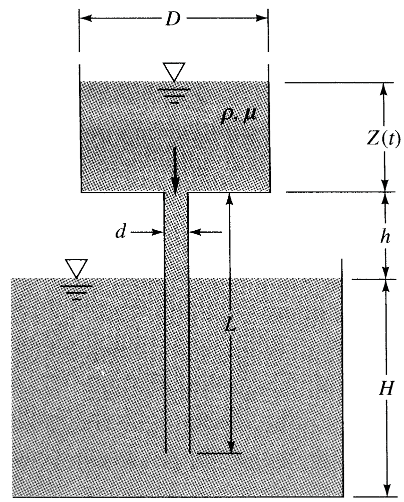

Fig. P6.27

Solution: The energy equation of Prob. 6.25, using symbols only, is combined with a control-volume mass balance for the tank to give the basic differential equation for Z(t):

$$\text{energy: } h_f = \frac{32\mu LV}{\rho g d^2} = h + Z; \quad \text{mass balance: } \frac{d}{dt}\left[\frac{\pi}{4}D^2 Z + \frac{\pi}{4}d^2 L\right] = -Q = -\frac{\pi}{4}d^2 V,$$

$$\text{or: } \frac{\pi}{4}D^2\frac{dZ}{dt} = -\frac{\pi}{4}d^2 V, \quad \text{where } V = \frac{\rho g d^2}{32\mu L}(h+Z)$$

Separate the variables and integrate, combining all the constants into a single "C":

$$\int_{Z_o}^{Z}\frac{dZ}{h+Z} = -C\int_0^t dt, \quad \text{or: } \mathbf{Z = (h + Z_o)e^{-Ct} - h}, \quad \text{where } \mathbf{C = \frac{\rho g d^4}{32\mu L D^2}} \quad \textit{Ans.}$$

$$\text{Tank drains completely when } Z = 0, \text{ at } \mathbf{t_0 = \frac{1}{C}\ln\left(1 + \frac{Z_o}{h}\right)} \quad \textit{Ans.}$$