where $h_f = \dfrac{32\mu LV}{\rho g d^2} = \dfrac{32(0.29)(25)(4.76)}{891(9.81)(0.03)^2} = 140.5\ m,$ Solve for $h_{pump} = 118.9\ m$

The pump power is then given by

$$\textbf{Power} = \rho g Q h_p = \dot{m} g h_p = \left(3\ \frac{kg}{s}\right)\left(9.81\ \frac{m}{s^2}\right)(118.9\ m) = \mathbf{3500\ watts} \quad \textit{Ans.}$$

6.34 Derive the time-averaged *x*-momentum equation (6.21) by direct substitution of Eqs. (6.19) into the momentum equation (6.14). It is convenient to write the convective acceleration as

$$\frac{du}{dt} = \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw)$$

which is valid because of the continuity relation, Eq. (6.14).

Solution: Into the *x*-momentum eqn. substitute u = u + u', v = v + v', etc., to obtain

$$\rho\left[\frac{\partial}{\partial x}(\bar{u}^2 + 2\bar{u}u' + u'^2) + \frac{\partial}{\partial y}(\bar{v}\,\bar{u} + \bar{v}u' + v'\bar{u} + v'u') + \frac{\partial}{\partial z}(\bar{w}\bar{u} + \bar{w}u' + w'\bar{u} + w'u')\right]$$

$$= -\frac{\partial}{\partial x}(\bar{p} + p') + \rho g_x + \mu[\nabla^2(\bar{u} + u')]$$

Now take the time-average of the entire equation to obtain Eq. (6.21) of the text:

$$\rho\left[\frac{d\bar{\mathbf{u}}}{\mathbf{dt}} + \frac{\partial}{\partial \mathbf{x}}(\overline{\mathbf{u'^2}}) + \frac{\partial}{\partial \mathbf{y}}(\overline{\mathbf{u'v'}}) + \frac{\partial}{\partial \mathbf{z}}(\overline{\mathbf{u'w'}})\right] = -\frac{\partial \bar{\mathbf{p}}}{\partial \mathbf{x}} + \rho \mathbf{g_x} + \mu\nabla^2(\bar{\mathbf{u}}) \quad \textit{Ans.}$$

6.35 By analogy with Eq. (6.21) write the turbulent mean-momentum differential equation for (a) the *y* direction and (b) the *z* direction. How many turbulent stress terms appear in each equation? How many unique turbulent stresses are there for the total of three directions?

Solution: You can re-derive, as in Prob. 6.34, or just permute the axes:

$$\text{(a)}\quad \mathbf{y}:\ \rho\frac{d\bar{v}}{dt} = -\frac{\partial \bar{p}}{\partial y} + \rho g_y + \frac{\partial}{\partial x}\left(\mu\frac{\partial \bar{v}}{\partial x} - \rho u'v'\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial \bar{v}}{\partial y} - \rho v'v'\right)$$

$$+ \frac{\partial}{\partial z}\left(\mu\frac{\partial \bar{v}}{\partial z} - \rho v'w'\right)$$