The match-point at the center gives us a log-law estimate of the shear stress:

$$\frac{V}{2u^*} \approx \frac{1}{\kappa}\ln\left(\frac{hu^*}{2\nu}\right) + \text{B}, \quad \kappa \approx 0.41,\ \text{B} \approx 5.0,\ \boldsymbol{u^* = (\tau_w/\rho)^{1/2}} \quad \textit{Ans.}$$

This is one form of "dimensionless shear stress." The more normal form is friction coefficient versus Reynolds number. Calculations from the log-law fit a Power-law curve-fit expression in the range $2000 < \text{Re}_h < 1\text{E}5$:

$$\mathbf{C_f} = \frac{\tau_w}{(1/2)\rho V^2} \approx \frac{0.018}{(\rho V h/\nu)^{1/4}} = \frac{\mathbf{0.018}}{\mathbf{Re_h^{1/4}}} \quad \textit{Ans.}$$

6.38 Suppose in Fig. P6.37 that h = 3 cm, the fluid is water at 20°C (ρ = 998 kg/m^3, μ = 0.001 kg/m·s), and the flow is turbulent, so that the logarithmic law is valid. If the shear stress in the fluid is 15 Pa, estimate V in m/s.

Solution: Just as in Prob. 6.37, apply the log-law at the center between the wall, that is, $y = h/2$, $u = V/2$. With τ_w known, we can evaluate u^* immediately:

$$u^* = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{15}{998}} = 0.123\ \frac{m}{s}, \quad \frac{V/2}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{u^* h/2}{\nu}\right) + B,$$

$$\text{or:}\quad \frac{V/2}{0.123} = \frac{1}{0.41}\ln\left[\frac{0.123(0.03/2)}{0.001/998}\right] + 5.0 = 23.3, \quad \textit{Solve for}\ \mathbf{V \approx 5.72\ \frac{m}{s}} \quad \textit{Ans.}$$

6.39 By analogy with laminar shear, $\tau = \mu\, du/dy$. T. V. Boussinesq in 1877 postulated that turbulent shear could also be related to the mean-velocity gradient $\tau_{turb} = \varepsilon\, du/dy$, where ε is called the *eddy viscosity* and is much larger than μ. If the logarithmic-overlap law, Eq. (6.28), is valid with $\tau \approx \tau_w$, show that $\varepsilon \approx \kappa\rho u^* y$.

Solution: Differentiate the log-law, Eq. (6.28), to find du/dy, then introduce the eddy viscosity into the turbulent stress relation:

$$\textit{If}\ \frac{u}{u^*} = \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B, \quad \textit{then}\ \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\textit{Then, if}\quad \tau \approx \tau_w \equiv \rho u^{*2} = \varepsilon\frac{du}{dy} = \varepsilon\frac{u^*}{\kappa y}, \quad \textit{solve for}\ \boldsymbol{\varepsilon = \kappa\rho u^* y} \quad \textit{Ans.}$$
