6.40 Theodore von Kármán in 1930 theorized that turbulent shear could be represented by $\tau_{turb} = \varepsilon\, du/dy$ where $\varepsilon = \rho\kappa^2 y^2 |du/dy|$ is called the *mixing-length eddy viscosity* and $\kappa \approx 0.41$ is Kármán's dimensionless *mixing-length constant* [2,3]. Assuming that $\tau_{turb} \approx \tau_w$ near the wall, show that this expression can be integrated to yield the logarithmic-overlap law, Eq. (6.28).

Solution: This is accomplished by straight substitution:

$$\tau_{turb} \approx \tau_w = \rho u^{*2} = \varepsilon \frac{du}{dy} = \left[\rho\kappa^2 y^2 \left|\frac{du}{dy}\right|\right]\frac{du}{dy}, \quad \text{solve for } \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Integrate: } \int du = \frac{u^*}{\kappa}\int \frac{dy}{y}, \quad \text{or: } \mathbf{u = \frac{u^*}{\kappa}\ln(y) + constant} \quad \textit{Ans.}$$

To convert this to the exact form of Eq. (6.28) requires fitting to experimental data.

P6.41 Two reservoirs, which differ in surface elevation by 40 m, are connected by 350 m of new pipe of diameter 8 cm. If the desired flow rate is at least 130 N/s of water at 20°C, may the pipe material be (a) galvanized iron, (b) commercial steel, or (c) cast iron? Neglect minor losses.

Solution: Applying the extended Bernoulli equation between reservoir surfaces yields

$$\Delta z = 40\,m = f\frac{L}{D}\frac{V^2}{2g} = f\left(\frac{350\,m}{0.08\,m}\right)\frac{V^2}{2(9.81\,m/s^2)}$$

where f and V are related by the friction factor relation:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right) \quad \text{where} \quad Re_D = \frac{\rho VD}{\mu}$$

When V is found, the weight flow rate is given by $w = \rho g Q$ where $Q = AV = (\pi D^2/4)V$. For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. Given the desired $w = 130$ N/s, solve this system of equations by EES to yield the wall roughness. The results are:

$f = 0.0257$; $V = 2.64$ m/s ; $Re_D = 211{,}000$; $\varepsilon_{max} = 0.000203$ m = **0.203 mm**

Any less roughness is OK. From Table 6-1, the three pipe materials have
(a) galvanized: $\varepsilon = 0.15$ mm ; (b) commercial steel: $\varepsilon = 0.046$ mm ; cast iron: $\varepsilon = 0.26$ mm

Galvanized and steel are fine, but cast iron is too rough.. *Ans.* Actual flow rates are

(a) galvanized: 135 N/s; (b) steel: 152 N/s; (c) cast iron: 126 N/s (*not enough*)

6.42 It is clear by comparing Figs. 6.12*b* and 6.13 that the effects of sand roughness and commercial (manufactured) roughness are not quite the same. Take the special case of commercial roughness ratio $\varepsilon/d = 0.001$ in Fig. 6.13, and replot in the form of the wall-law shift ΔB (Fig. 6.12*a*) versus the logarithm of $\varepsilon^+ = \varepsilon u^*/\nu$. Compare your plot with Eq. (6.45).

Solution: To make this plot we must relate ΔB to the Moody-chart friction factor. We use Eq. (6.33) of the text, which is valid for any B, in this case, $B = B_o - \Delta B$, where $B_o \approx 5.0$:

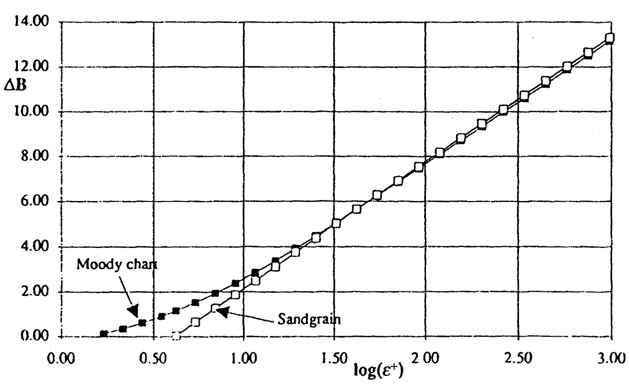
$$\frac{V}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{Ru^*}{\nu}\right) + B_o - \Delta B - \frac{3}{2\kappa}, \quad \text{where} \quad \frac{V}{u^*} = \sqrt{\frac{8}{f}} \quad \text{and} \quad \frac{Ru^*}{\nu} = \frac{1}{2}Re_d\sqrt{\frac{f}{8}} \qquad (1)$$

Combine Eq. (1) with the Colebrook friction formula (6.48) and the definition of ε^+:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \qquad (2)$$

$$\text{and} \quad \varepsilon^+ = \frac{\varepsilon u^*}{\nu} = \frac{\varepsilon}{d}d^+ = \frac{\varepsilon}{d}Re\sqrt{\frac{f}{8}} \qquad (3)$$

Equations (1, 2, 3) enable us to make the plot below of "commercial" log-shift ΔB, which is similar to the 'sand-grain' shift predicted by Eq. (6.45): $\Delta B_{sand} \approx (1/\kappa)\ln(\varepsilon^+) - 3.5$.



Ans.

Fig. P6.42
