6.71 It is desired to solve Prob. 6.62 for the most economical pump and cast-iron pipe system. If the pump costs \$125 per horsepower delivered to the fluid and the pipe costs \$7000 per inch of diameter, what are the minimum cost and the pipe and pump size to maintain the 3 ft^3/s flow rate? Make some simplifying assumptions.

Solution: For water at 20°C, take $\rho = 1.94$ $slug/ft^3$ and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft. Write the energy equation (from Prob. 6.62) in terms of Q and d:

$$P_{\text{in hp}} = \frac{\rho g Q}{550}(\Delta z + h_f) = \frac{62.4(3.0)}{550}\left\{120 + f\left(\frac{2000}{d}\right)\frac{[4(3.0)/\pi d^2]^2}{2(32.2)}\right\} = 40.84 + \frac{154.2f}{d^5}$$

$$\text{Cost} = \$125P_{hp} + \$7000d_{inches} = 125(40.84 + 154.2f/d^5) + 7000(12d), \quad \text{with d in ft.}$$

$$\text{Clean up:} \quad \text{Cost} \approx \$5105 + 19278f/d^5 + 84000d$$

Regardless of the (unknown) value of f, this Cost relation does show a minimum. If we assume for simplicity that *f* is constant, we may use the differential calculus:

$$\frac{d(\text{Cost})}{d(d)}\bigg|_{f\approx\text{const}} = \frac{-5(19278)f}{d^6} + 84000, \quad \text{or} \quad d_{best} \approx (1.148\ f)^{1/6}$$

$$\text{Guess } f \approx 0.02, \quad d \approx [1.148(0.02)]^{1/6} \approx 0.533 \text{ ft}, \quad Re = \frac{4\rho Q}{\pi\mu d} \approx 665000, \quad \frac{\varepsilon}{d} \approx 0.00159$$

$$\text{Then} \quad f_{better} \approx 0.0224, \quad d_{better} \approx 0.543 \text{ ft (converged)}$$

Result: $d_{best} \approx 0.543$ ft ≈ **6.5 in,** $Cost_{min} \approx \$14300_{pump} + \$45600_{pipe}$ ≈ **\$60000**. *Ans.*

6.72 Modify Prob. P6.57 by letting the diameter be unknown. Find the proper pipe diameter for which the pool will drain in about 2 hours flat.

Solution: Recall the data: Let $W = 5$ m, $Y = 8$ m, $h_o = 2$ m, $L = 15$ m, and $\varepsilon = 0$, with water, $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. We apply the same theory as Prob. 6.57:

$$V = \sqrt{\frac{2gh}{1 + fL/D}}, \quad t_{drain} \approx \frac{4WY}{\pi D^2}\sqrt{\frac{2h_o(1 + f_{av}L/D)}{g}}, \quad f_{av} = fcn(Re_D) \quad \text{for a smooth pipe.}$$

For the present problem, $t_{drain} = 2$ hours and D is the unknown. Use an average value $h = 1$ m to find f_{av}. Enter these equations on EES (or you can iterate by hand) and the final results are

$$V = 2.36 \text{ m/s}; \quad Re_D = 217{,}000; \quad f_{av} \approx 0.0154; \quad D = 0.092 \text{ m} \approx \mathbf{9.2\ cm} \quad \textit{Ans.}$$