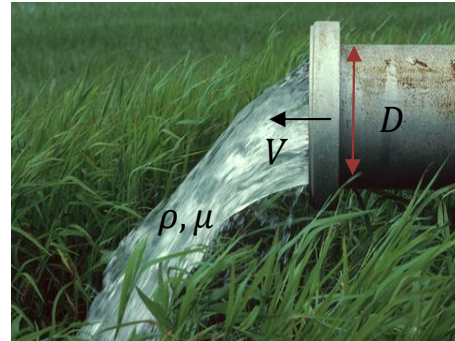


ENSC 283 Week # 2, Tutorial # 1– Dimensionless Quantities

Problem 1: A dimensionless combination of variables that is important in the study of viscous flow through pipes is called the *Reynolds number*, Re , defined as $\rho VD/\mu$ where, as indicated in the figure, ρ is the fluid density, V the mean fluid velocity, D the pipe diameter, and μ the fluid viscosity. A Newtonian fluid having a viscosity of $0.38 \text{ N}\cdot\text{s}/\text{m}^2$ and a specific gravity of 0.91 flows through a 25-mm-diameter pipe with a velocity of $2.6 \text{ m}/\text{s}$. Determine the value of the Reynolds number using (a) SI units, and (b) BG units.



Solution

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

- Reynolds number, Re , in SI and BG units

Step 2: Prepare a data table

| Data | Value | Unit |
|-------|-------|------------------------------------|
| V | 2.6 | m/s |
| D | 25 | mm |
| μ | 0.38 | $\text{N}\cdot\text{s}/\text{m}^2$ |
| SG | 0.91 | - |

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

- 1) Water is considered as a Newtonian fluid.

Step 4: Calculations

(a) SI Units

The fluid density is calculated from the specific gravity as

$$\rho = SG \rho_{H_2O@4^\circ C} = 0.91 \left(1000 \frac{kg}{m^3} \right) = 910 \text{ kg}/m^3 \quad (\text{Eq1})$$

From the definition of the Reynolds number,

$$\begin{aligned} Re &= \frac{\rho V D}{\mu} = \frac{\left(910 \frac{kg}{m^3} \right) \left(2.6 \frac{m}{s} \right) (25 \text{ mm}) (10^{-3} \frac{mm}{m})}{0.38 \text{ N}\cdot\text{s}/m^2} \quad (\text{Eq2}) \\ &= 156 \left(kg \cdot \frac{m}{s^2} \right) / N \end{aligned}$$

However, since $1N = 1 \text{ kg}\cdot\text{m}/s^2$ it follows that the Reynolds number is unitless, therefore

$$Re = 156 \quad (\text{Eq3})$$

(b) BG Units

We first convert all the SI values of the variables appearing in the Reynolds number to BG values by using the conversion factors.

Since $1kW = 1kJ/s$, then the maximum electrical power generation per year become

$$\rho = \left(910 \frac{kg}{m^3} \right) \left(1.940 \times 10^{-3} \frac{\frac{slugs}{ft^3}}{\frac{kg}{m^3}} \right) = 1.77 \text{ slugs}/ft^3 \quad (\text{Eq4})$$

$$V = \left(2.6 \frac{m}{s} \right) \left(3.281 \frac{ft}{m} \right) = 8.53 \text{ ft}/s \quad (\text{Eq5})$$

$$D = (0.025 \text{ m}) \left(3.281 \frac{\text{ft}}{\text{m}} \right) = 8.20 \times 10^{-2} \text{ ft} \quad (\text{Eq6})$$

$$\mu = (0.38 \text{ N}\cdot\text{s}/\text{m}^2) \left(2.089 \frac{\text{lb}\cdot\text{s}/\text{ft}^2}{\text{N}\cdot\text{s}/\text{m}^2} \right) = 7.94 \times 10^{-3} \text{ lb}\cdot\text{s}/\text{ft}^2 \quad (\text{Eq7})$$

Using the definition of the Reynolds number, we get

$$Re = \frac{\rho V D}{\mu} = \frac{\left(1.77 \frac{\text{slugs}}{\text{ft}^3} \right) \left(8.53 \frac{\text{ft}}{\text{s}} \right) (8.20 \times 10^{-2} \text{ ft})}{7.94 \times 10^{-3} \text{ lb}\cdot\text{s}/\text{ft}^2} = 156 \quad (\text{Eq8})$$

Step 5: Concluding Statement

The value of any dimensionless quantity does not depend on the system of units used if all variables that make up the quantity are expressed in a consistent set of units.