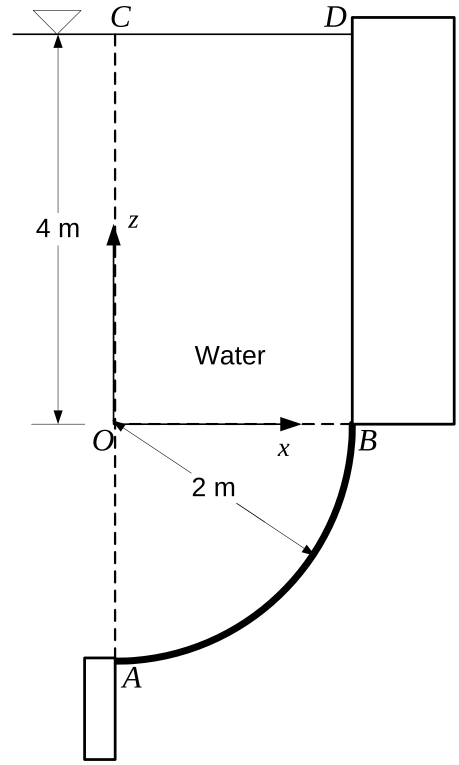
ENSC 283 Week # 4, Tutorial # 3– Hydrostatic Forces

Problem 1: In the figure the surface *AB* is a circular arc with a radius of *2 m*. The distance *DB* is *4 m*. If water is the liquid supported by the surface and if atmospheric pressure prevails on the other side of *AB*, determine the magnitude and line of action of the resultant hydrostatic force on *AB* per unit length.



Solution

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

– the magnitude and line of action of the hydrostatic force *F* on *AB*

Step 2: Prepare a data table

Data	Value	Unit
DB	4	m
r	2	m

Step 3: Calculations

The vertical component is equal to the weight of water in volume *AOCDB*:

$$W_{OCDB} = \gamma\, A_{OCDB} \times 1 = \left(9810 \frac{N}{m^3}\right)(4\ m)(2\ m)(1\ m) = 78.480\ kN \qquad \text{(Eq1)}$$

$$W_{AOB} = \gamma\, A_{AOB} \times 1 = \gamma \left(\frac{1}{4}\pi r^2\right) \times 1 = \left(9810\ \frac{N}{m^3}\right) \times \frac{\pi}{4}(2\ m)^2(1\ m) = 30.819\ kN \qquad \text{(Eq2)}$$

Therefore, the vertical component is:

$$F_V = W_{OCDB} + W_{AOB} = 109.299\ kN \qquad \text{(Eq3)}$$

The line of action of the vertical component acts through the centroid of the volume of water considered above, and this is calculated by taking moments about the *z*-axis.

$$\begin{aligned} F_V . x_{CG} &= W_{OCDB} . x_{CG,1} + W_{AOB} . x_{CG,2} \\ &= (78.480\ kN)(1\ m) + (30.819\ \ kN)\left(\frac{4 \times 2}{3\pi}\ m\right) \\ &= 104.640\ kNm \end{aligned} \qquad \text{(Eq4)}$$

$$x_{CG} = \frac{104.640\ kNm}{109.299\ kN} = 0.9574\ m \qquad \text{(Eq5)}$$

Note: $x_{CG,1}$ and $x_{CG,2} = 4r/(3\pi)$ are the centroidal distances of $OCDB$ and AOB with respect to the *z*-axis, respectively.

The magnitude of the horizontal component can be expressed as:

$$F_H = \gamma h_{CG} A_{proj} = \left(9810 \frac{N}{m^3}\right)(5\ m)(2\ m^2) = 98.1\ kN \qquad \text{(Eq6)}$$

The location of the line of action of the horizontal component is given by:

$$y_{CP,proj} = -\frac{I_{xx}\sin\theta}{h_{CG}A_{proj}} = -\frac{\left(\frac{1}{12}\right)(1\,m)(2\,m)^3\sin 90°}{(5\,m)(2\,m^2)} = -0.0667\,m \qquad \text{(Eq7)}$$

Note that $y_{CP,proj}$ is calculated with respect to the center of A_{proj}, therefore,

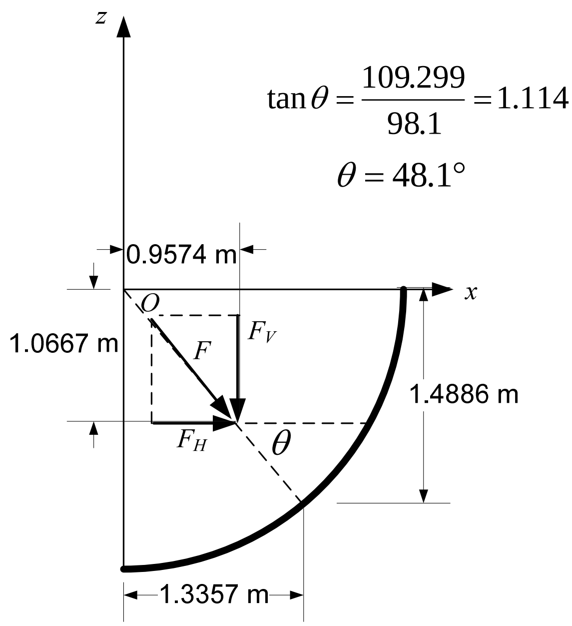
$$Y_{CP,proj} = -1\,m - 0.0667\,m = -1.0667\,m \qquad \text{(Eq8)}$$

where, $Y_{CP,proj}$ is the location with respect to the *x*-axis.

The resultant hydrostatic force is:

$$F = (F_H^2 + F_V^2)^{1/2} = [(98.1\,kN)^2 + (109.299\,kN)^2]^{1/2} = 146.867\,kN \qquad \text{(Eq9)}$$

This resultant force is shown in the following figure.



It should be considered that the resultant force acts along $z = -(\tan\theta)\,x$.