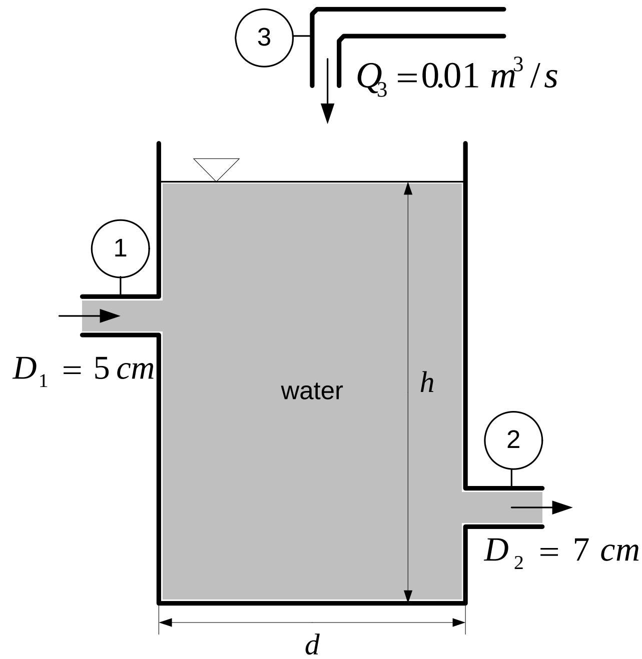
ENSC 283 Week # 5, Tutorial # 4 – Conservation of Mass

Problem 1: The open tank in the figure contains water at *20°C*. For incompressible flow, (a) derive an analytic expression for dh/dt in terms of (Q_1, Q_2, Q_3). (b) If h is constant, determine V_2 for the given data if $V_1 = 3\ m/s$ and $Q_3 = 0.01\ m^3/s$.



Solution

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

- dh/dt as a function of Q_1, Q_2, Q_3
- V_2

Step 2: Prepare a data table

Data	Value	Unit
Q_3	0.01	m^3/s
V_1	3	m/s
D_1	5	cm
D_2	7	cm

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) Incompressible flow ($\rho = Constant$)
2) The tank and pipes have circular cross-sections

Step 4: Calculations

(a) For a control volume enclosing the tank, conservation of mass can be expressed as:

$$\frac{d}{dt}\left(\int_{CV} \rho \, d\vartheta\right) + \rho(Q_2 - Q_1 - Q_3) = 0 \qquad \text{(Eq1)}$$

The volume of the tank is:

$$\vartheta = \frac{\pi d^2}{4} h \qquad \text{(Eq2)}$$

Substituting Eq2 into Eq1, we get:

$$\rho \frac{\pi d^2}{4} \frac{dh}{dt} + \rho(Q_2 - Q_1 - Q_3) = 0 \qquad \text{(Eq3)}$$

Finally,

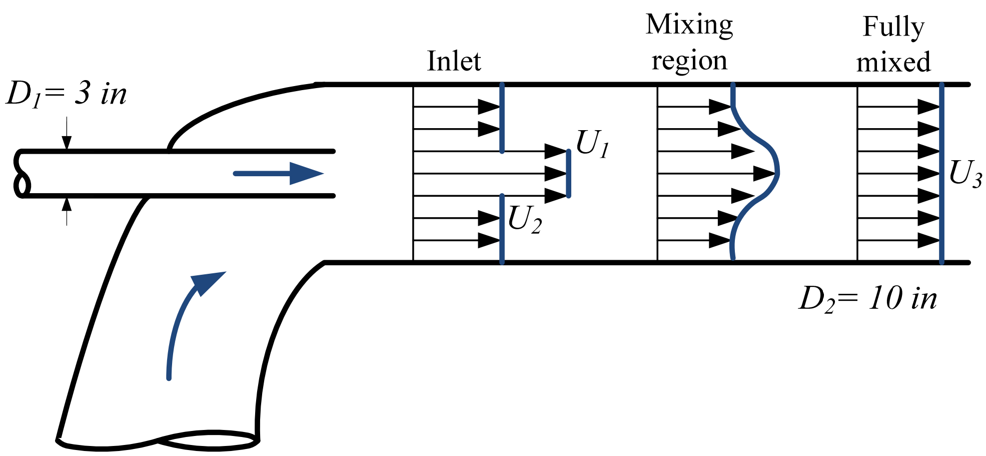
$$\boldsymbol{\frac{dh}{dt} = \frac{Q_1 + Q_3 - Q_2}{(\pi d^2/4)}} \qquad \text{(Eq4)}$$

(b) If *h* is constant, then

$$\frac{dh}{dt} = 0 = Q_2 - Q_1 - Q_3 \longrightarrow Q_2 = Q_1 + Q_3 \rightarrow \frac{\pi}{4}(0.07)^2(V_2) = 0.01 + \frac{\pi}{4}(0.05)^2(3) \qquad \text{(Eq5)}$$

$$\boldsymbol{V_2 = 4.13 \ m/s} \qquad \text{(Eq6)}$$

Problem 2: The jet pump in the figure injects water at $U_1 = 40\ m/s$ through a 3-in pipe and entrains a secondary flow of water $U_2 = 3\ m/s$ in the annular region around the small pipe. The two flows become fully mixed downstream, where U_3 is approximately constant. For steady incompressible flow, compute U_3 in *m/s*.



<u>**Solution**</u>

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

– The velocity in fully mixed region, U_3 in *m/s*

Step 2: Prepare a data table

Data	Value	Unit
D_1	3	in
D_2	10	in
U_1	40	m/s
U_2	3	m/s

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) Steady incompressible flow

Step 4: Calculations

First modify the units:

$$D_1 = (3\ in)\left(\frac{2.54 \times 10^{-2} m}{1\ in}\right) = 0.0762\ m \tag{Eq1}$$

$$D_2 = (10\ in)\left(\frac{2.54 \times 10^{-2} m}{1\ in}\right) = 0.254\ m \tag{Eq2}$$

For incompressible flow, the volume flows at inlet and exit must match:

$$Q_1 + Q_2 = Q_3 \rightarrow \frac{\pi}{4} {D_1}^2 U_1 + \frac{\pi}{4}\left[{D_2}^2 - {D_1}^2\right] U_2 = \frac{\pi}{4} {D_2}^2 U_3 \tag{Eq3}$$
$$\rightarrow \frac{\pi}{4}(0.0762)^2(40) + \frac{\pi}{4}[(0.254)^2 - (0.0762)^2](3) = \frac{\pi}{4}(0.254)^2 U_3$$

Solving the above equation, we get:

$$\boldsymbol{U_3 = 6.33\ m/s} \tag{Eq4}$$