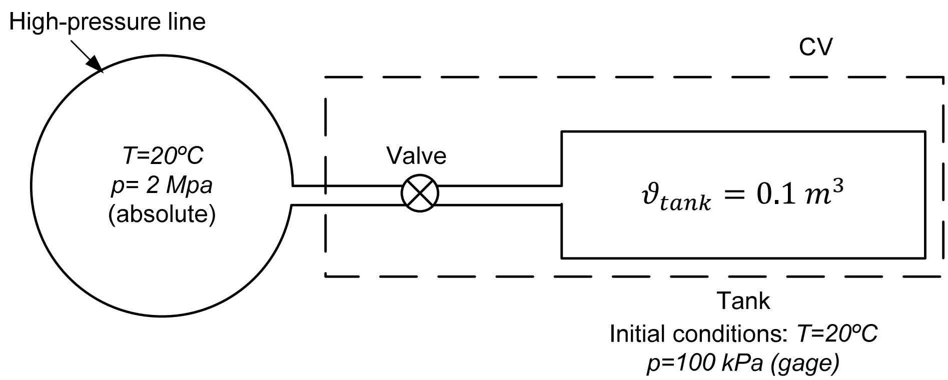
ENSC 283 Week # 8, Tutorial # 5 – First Law Analysis

Problem 1: A tank of $0.1\ m^3$ volume is connected to a high-pressure air line; both line and tank are initially at a uniform temperature of $20℃$. The initial tank gage pressure is $100\ kPa$. The absolute line pressure is $2.0\ MPa$; the line is large enough so that its temperature and pressure may assumed constant. The tank temperature is monitored by a fast-response thermocouple. At the instant after the valve is opened, the tank temperature rises at the rate of $0.05℃/s$. Determine the instantaneous flow rate of air into the tank if heat transfer is neglected.



<u>Solution</u>

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

– $\dot{m}$, the instantaneous flow rate of air into the tank

Step 2: Prepare a data table

Data	**Value**	**Unit**
ϑ_{tank}	0.1	m^3
$p_{tank}\ (gage)$	100	kPa
T	20	℃
$\dot{m}$	0.05	$℃/s$

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) $\dot{Q} = 0$
2) $\dot{W}_s = 0$
3) $\dot{W}_v = 0$
4) $\dot{W}_{other} = 0$
5) Velocities in line and tank are small
6) Neglect potential energy
7) Properties uniform in tank
8) Ideal gas, $p = \rho RT$,$d\hat{u} = c_v dT$

Step 4: Calculations

The control volume is shown in the figure. Applying the energy equation to the CV, we get:

$$\dot{Q} - \dot{W}_s - \dot{W}_v - \dot{W}_{other} = \frac{\partial}{\partial t}\left(\int_{CV} e\,\rho\,d\vartheta\right) + \int_{CS} (e + pv)\,\rho\,(\boldsymbol{V} \cdot \boldsymbol{n})dA \qquad \text{(Eq1)}$$

The left hand side of the above equation is zero, see the assumptions. The total energy can be written as:

$$e = \hat{u} + \frac{V^2}{2} + gz = \hat{u} \qquad \text{(Eq2)}$$

Applying the assumptions and substituting Eq2 into Eq1, we obtain:

$$\frac{\partial}{\partial t}\left(\int_{CV} \hat{u}_{tank}\,\rho\,d\vartheta\right) + (\hat{u} + pv)_{line}(-\rho VA) = 0 \qquad \text{(Eq3)}$$

This expresses the fact the gain in energy of the tank is due to influx of fluid energy (in the form of enthalpy $h = \hat{u} + pv$) from the line. We are interested in the initial instant, when T is uniform at $20℃$, so $\hat{u}_{tank} = \hat{u}_{line} = \hat{u}$, the internal energy at T; also, $pv_{line} = RT$, and

$$\frac{\partial}{\partial t}\left(\int_{CV} \hat{u}\,\rho\,d\vartheta\right) + (\hat{u} + RT)(-\rho VA) = 0 \qquad \text{(Eq4)}$$

Since tank properties are uniform, $\partial/\partial t$ may be replaced by d/dt, and

$$\frac{d}{dt}(\hat{u}M) = (\hat{u} + RT)\dot{m} \qquad \text{(Eq5)}$$

where M is the instantaneous mass in the tank and $\dot{m} = \rho VA$ is the mass flow rate. This equation can be written as:

$$\hat{u}\frac{dM}{dt} + M\frac{d\hat{u}}{dt} = \hat{u}\dot{m} + RT\dot{m} \qquad \text{(Eq6)}$$

The term dM/dt may be evaluated from continuity:

$$\frac{\partial}{\partial t}\left(\int_{CV} \rho\,d\vartheta\right) + \int_{CS} \rho\,(\boldsymbol{V}\cdot\boldsymbol{n})dA = 0 \qquad \text{(Eq7)}$$

$$\frac{dM}{dt} + (-\rho VA) = 0 \;or\; \frac{dM}{dt} = \dot{m} \qquad \text{(Eq8)}$$

Substituting in Eq6 gives

$$\hat{u}\dot{m} + Mc_v\frac{dT}{dt} = \hat{u}\dot{m} + RT\dot{m} \qquad \text{(Eq9)}$$

or

$$\dot{m} = \frac{Mc_v\frac{dT}{dt}}{RT} = \frac{\rho_{tank}\vartheta_{tank}c_v\frac{dT}{dt}}{RT} \qquad \text{(Eq10)}$$

At $t = 0$, $p_{tank} = 100\;kPa\;(gage)$, and

$$\rho_{tank} = \frac{p_{tank}}{RT} = (1 + 1.0135)10^5\,\frac{N}{m^2} \times \frac{kg.K}{287\;N.m} \times \frac{1}{293.15\;K} = 2.393\;kg/m^3 \qquad \text{(Eq11)}$$

Substituting into Eq10, we obtain

$$\dot{m} = \left(2.393\ \frac{kg}{m^3}\right)(0.1\ m^3)\left(717\ \frac{N.m}{kg.K}\right)\left(0.05\ \frac{K}{s}\right)\left(\frac{kg.K}{287\ N.m}\right) \times \left(\frac{1}{293.15\ K}\right)\left(\frac{1000\ g}{kg}\right) = \mathbf{0.102\ g/s} \qquad \text{(Eq12)}$$