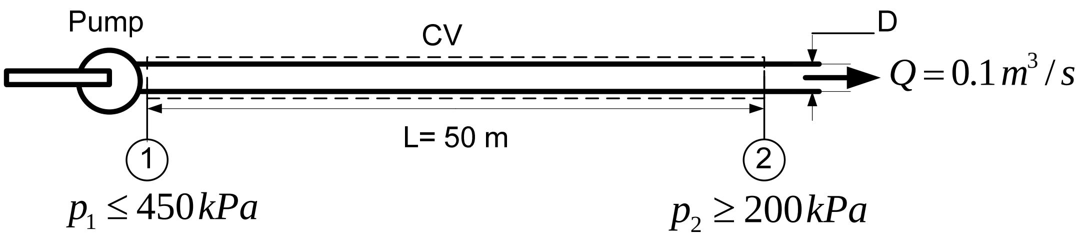
ENSC 283 Week # 12, Tutorial # 8 – Flow in an Irrigation System

Problem: Spray heads in an agricultural spraying system are to be supplied with water through $50\ m$ of drawn aluminum tubing from an engine-driven pump. In its most efficient operating range, the pump output is $0.1\ m^3/s$ at a discharge pressure not exceeding $450\ kPa$. For satisfactory operation, the sprinklers must operate at $200\ kPa$ or higher pressure. Minor losses and elevation changes may be neglected. Assuming the roughness of $0.01 mm$ for the pipe, determine the smallest standard pipe size that can be used.



Solution

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

– D, the smallest standard pipe size

Step 2: Prepare a data table

Data	Value	Unit
Q	0.1	m^3/s
p_{1max}	450	kPa
p_{2min}	200	kPa
L	50	m
ϵ	0.01	mm

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) Steady state
2) Incompressible flow
3) Negligible minor losses i.e., $h_f = h_{pump}$
4) $\bar{V}_1 = \bar{V}_2 = \bar{V}; \alpha_1 = \alpha_2$

Step 4: Calculations

Δp, L, and Q are known. D is unknown, so iteration is needed to determine the minimum standard diameter that satisfies the pressure drop constraint at the given flow rate. The maximum allowable pressure drop over the length, L, is

$$\Delta p_{max} = p_{1max} - p_{2min} = (450 - 200)kPa = 250kPa \qquad \text{(Eq1)}$$

Neglecting elevation changes and the energy equation can be expressed as:

$$\left(\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^{\,2}}{2g}\right) - \left(\frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^{\,2}}{2g}\right) = h_f \qquad \text{(Eq2)}$$

$$h_{pump} = h_f = f \frac{L}{D} \frac{\bar{V}^2}{2g} \qquad \text{(Eq3)}$$

Applying the assumption (4), we get

$$\Delta p = p_1 - p_2 = f \frac{L}{D} \frac{\rho \bar{V}^2}{2} \qquad \text{(Eq4)}$$

This equation is difficult to solve for D because both $\bar{V}$ and f depend on D. The best approach is to use a computer application such as Excel to automatically solve for D. Fore completeness here we show the manual iteration procedure. The first step is to express Eq4 and the Reynolds number in terms of Q instead of $\bar{V}$ (Q is constant but $\bar{V}$ varies with D). We have $\bar{V} = Q/A = 4Q/\pi D^2$, so that

$$\Delta p = f \frac{L}{D}\frac{\rho}{2}\left(\frac{4Q}{\pi D^2}\right)^2 \tag{Eq5}$$

The Reynolds number in terms of Q is

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{4Q}{\pi D \nu} = \frac{4(0.1\ m^3/s)}{\pi(10^{-6} m^2/s)} = \frac{1.273 \times 10^5}{D} \tag{Eq6}$$

For an initial guess, take nominal pipe 4 in pipe:

$$Re = \frac{1.273 \times 10^5}{(4\ in)}\left(\frac{1\ in}{2.54 \times 10^{-2} m}\right) = 1.253 \times 10^6 \tag{Eq7}$$

For the drawn tubing, $\epsilon = 0.01mm$ and hence $\frac{\epsilon}{D} = 9.84 \times 10^{-5}$. Using the Colebrook relationship, the friction factor can be found.

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \rightarrow f = 0.01318 \tag{Eq8}$$

Substituting f into Eq5, we get:

$$\Delta p = \frac{8fL\rho Q^2}{\pi^2 D^5} = \frac{8 \times 0.01318 \times (50\ m)(1000\ kg/m^3)(0.1\ m^3/s)^2}{\pi^2 (10.16 \times 10^{-2} m)^5} = 493.409\ kPa > \Delta p_{max} \tag{Eq9}$$

Since $\Delta p > \Delta p_{max}$, we should try a larger diameter pipe. Let consider $D = 5\ in$, thus

$$Re = \frac{1.273 \times 10^5}{(5\ in)}\left(\frac{1\ in}{2.54 \times 10^{-2} m}\right) = 1.002 \times 10^6 \tag{Eq10}$$

For the drawn tubing with $D = 5\ in$, $\frac{\epsilon}{D} = 7.87 \times 10^{-5}$. Using the Colebrook relationship, we get $f = 0.01312$. Substituting f into Eq5, we get:

$$\Delta p = \frac{8fL\rho Q^2}{\pi^2 D^5} = \frac{8 \times 0.01312 \times (50\ m)(1000\ kg/m^3)(0.1\ m^3/s)^2}{\pi^2 (12.7 \times 10^{-2} m)^5} = 160.944\ kPa < \Delta p_{max} \tag{Eq11}$$

Thus, the criterion for pressure drop is satisfied for a minimum nominal diameter of 5 in pipe.