

ENSC 388

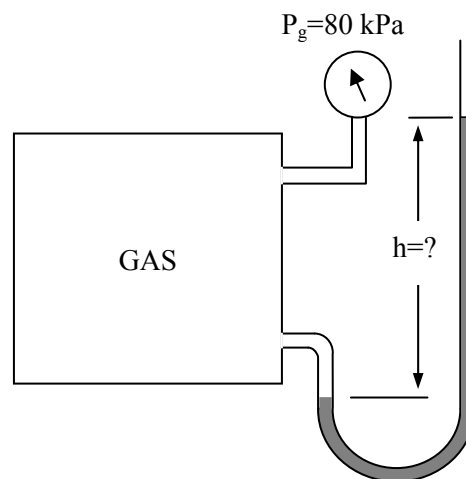
Assignment #1 (Basic Concepts)

Assignment date: Wednesday Sept. 16, 2009

Due date: Wednesday Sept. 23, 2009

Problem 1: (Static Pressure)

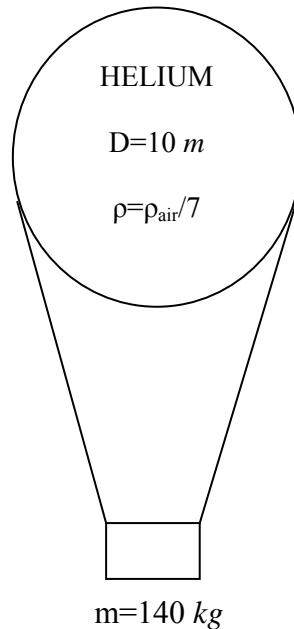
Both a gage and a manometer are attached to a gas tank to measure its pressure. If the reading on the pressure gage is 80 kPa , determine the distance between the two fluid levels of the manometer if the fluid is (a) mercury ($\rho = 13,600 \text{ kg/m}^3$) or (b) water ($\rho = 1000 \text{ kg/m}^3$).



Problem 2: (Buoyancy)

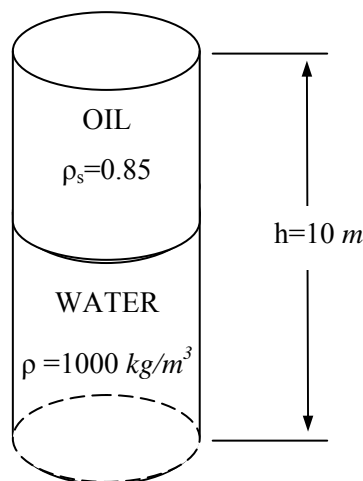
Balloons are often filled with helium gas because it weighs only about one-seventh of what air weighs under identical conditions. The buoyancy force which can be expressed as $F_B = \rho_{\text{air}} g V_{\text{balloon}}$ will push the balloon upward. If the balloon has diameter of 10 m and carries two people, 70 kg each, determine (a) the acceleration of the balloon when it is first released and (b) the maximum amount of load, in kg ,

the balloon can carry. Assume the density of air is $\rho = 1.16 \text{ kg/m}^3$, and neglect the weight of the ropes and the cage. (Answers: 16.5 m/s^2 , 520.6 kg)



Problem 3: (Hydrostatic pressure)

The lower half of a 10-m-high cylindrical container is filled with water ($\rho = 1000 \text{ kg/m}^3$) and the upper half with oil that has a specific gravity of 0.85. Determine the pressure difference between the top and bottom of the cylinder. (Answer: 90.7 kPa)

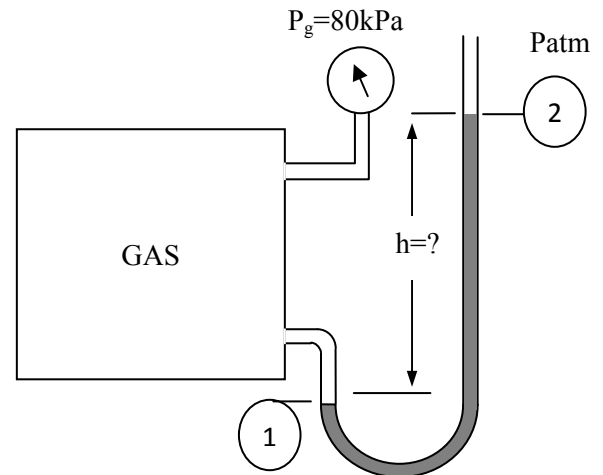


Problem 1:

$$P_g = 80 \text{ kPa}$$

Find **h** if:

- Fluid is mercury ($\rho_{Hg} = 13,600 \text{ kg/m}^3$)
- Fluid is water ($\rho_{H_2O} = 1000 \text{ kg/m}^3$)



- Assumption:

The pressure is uniform in the tank, thus we can determine the pressure at the gage port.

- Analysis

Starting with P_g (gas pressure) and moving along the tube from point (1) by adding (as we go down) or subtracting (as we go up), the ρgh term(s) until we reach point (2), therefore;

$$P_g - \rho gh = P_2$$

Since tube at point (2) is open to atmosphere, $P_2 = p_{atm}$.

$$\underbrace{P_g - P_2}_{P_{\text{gage}}} = \rho gh$$

P_{gage} (What we read on the pressure gage)

$$80 \text{ kPa} = \rho gh \Rightarrow \begin{cases} H_2O \rightarrow 80 \times 10^3 \text{ [Pa]} = 1000 \left[\frac{\text{kg}}{\text{m}^3} \right] \times 9.81 \left[\frac{\text{m}}{\text{s}^2} \right] h_{H_2O} \\ \Rightarrow h_{H_2O} = 8.155 \text{ [m]} \\ Hg \rightarrow 80 \times 10^3 \text{ [Pa]} = 13,600 \left[\frac{\text{kg}}{\text{m}^3} \right] \times 9.81 \left[\frac{\text{m}}{\text{s}^2} \right] h_{Hg} \\ \Rightarrow h_{Hg} = 0.599 \text{ [m]} \end{cases}$$

Problem 2:

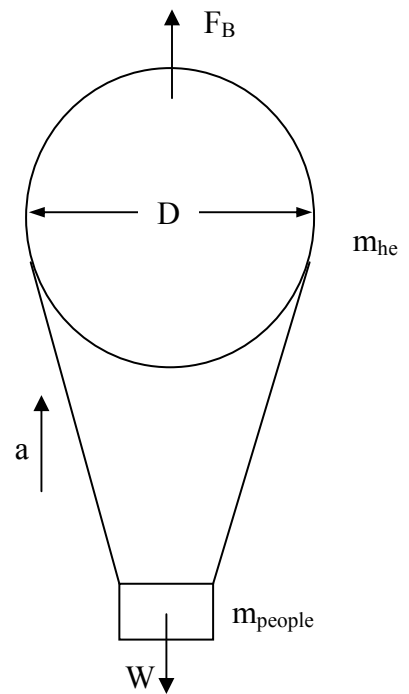
$$D=10 \text{ m}$$

$$\rho_{\text{He}} = \rho_{\text{air}}/7 \text{ and } \rho_{\text{air}} = 1.16 \text{ kg/m}^3$$

$$m_{\text{people}} = 140 \text{ kg}$$

Assumption:

The weight of the ropes and cage is neglected.



(Free body diagram)

Analysis:

Starting with free body diagram. We also know that the buoyancy force is:

$$F_B = \rho_{\text{air}} g V_{\text{balloon}}$$

$$W = m_{\text{total}} g$$

$$m_{\text{total}} = m_{\text{people}} + m_{\text{He}}$$

$$m_{\text{He}} = \rho_{\text{He}} V_{\text{balloon}}$$

$$V_{\text{balloon}} = \frac{4}{3} \pi R_{\text{balloon}}^3$$

$$V_{\text{balloon}} = \frac{4}{3} \pi (5[\text{m}])^3 = 523.59[\text{m}^3]$$

$$m_{\text{He}} = \frac{1}{7} \times 1.16 \left[\frac{\text{kg}}{\text{m}^3} \right] \times 523.59[\text{m}^3] = 86.77$$

$$m_{\text{total}} = m_{\text{He}} + m_{\text{people}} = 86.77[\text{kg}] + 140[\text{kg}] = 226.77[\text{kg}]$$

Newton law:

$$\sum F = m_{total} \times a$$

Therefore:

$$F_B - W = m_{total} \times a \quad (1)$$

$$\rho_{air} g V_{balloon} - m_{total} g = m_{total} a$$

So,

$$a = \frac{\rho_{air} V_{balloon} - m_{total}}{m_{total}} g \quad (2)$$

Substituting values in Eq. (2),

$$a = \frac{1.16 \left[\frac{kg}{m^3} \right] \times 523.59 [m^3] - 226.77 [kg]}{226.77 [kg]} \times 9.81 \left[\frac{m}{s^2} \right]$$

$$\Rightarrow \boxed{a = 16.46 \left[\frac{m}{s^2} \right]}$$

The maximum amount of load that the balloon can carry can be calculated from;

$$\sum F = 0$$

Using Eq. (1),

$$F_B = W$$

$$\rho_{air} V_{balloon} = m_{max}$$

$$m_{max} = \rho_{air} V_{balloon} = 1.16 \left[\frac{kg}{m^3} \right] \times 523.59 [m^3] = 607.304$$

This mass is including the mass of helium gas in the balloon. To calculate maximum load, mass of helium must subtracted from the maximum mass so,

$$\boxed{m_{max,Load} = m_{max} - m_{he} = 607.304 [kg] - 86.77 [kg] = 520.6 [kg]}$$

Problem 3:

$$\rho_{oil} / \rho_{H_2O} = 0.85$$

Analysis:

Starting with point (1) and adding terms ρgh as we go down,

$$P_1 + \rho_{oil}gh_{oil} + \rho_{H_2O}gh_{H_2O} = P_2$$

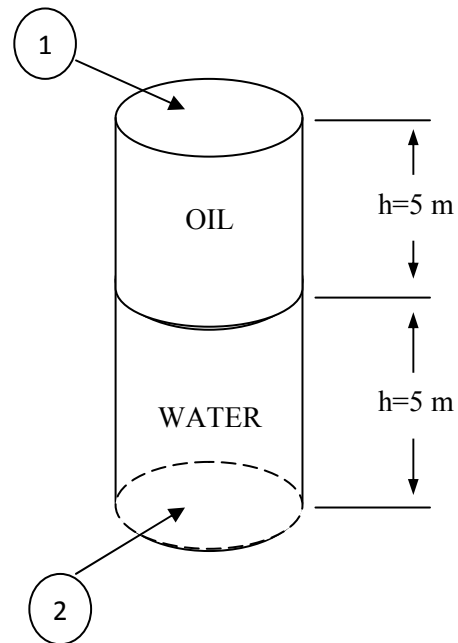
$$\rho_{oil}gh_{oil} + \rho_{H_2O}gh_{H_2O} = P_2 - P_1$$

$$P_2 - P_1 = \left(\rho_{oil} / \rho_{H_2O} h_{oil} + h_{H_2O} \right) \rho_{H_2O}g$$

Substituting values,

$$\rho_{H_2O} = 1000 \left[kg/m^3 \right]$$

$$\begin{aligned} P_2 - P_1 &= (0.85[-] \times 5[m] + 5[m])1000 \left[kg/m^3 \right] \times 9.81 \left[m/s^2 \right] \\ &= 90.742[kPa] \end{aligned}$$



Notes (1): Always substitute the numerical value at the last step.

Note (2): Write all the dimensions in your solution; check both sides of relationships for unit and dimension homogeneity.