ENSC 388

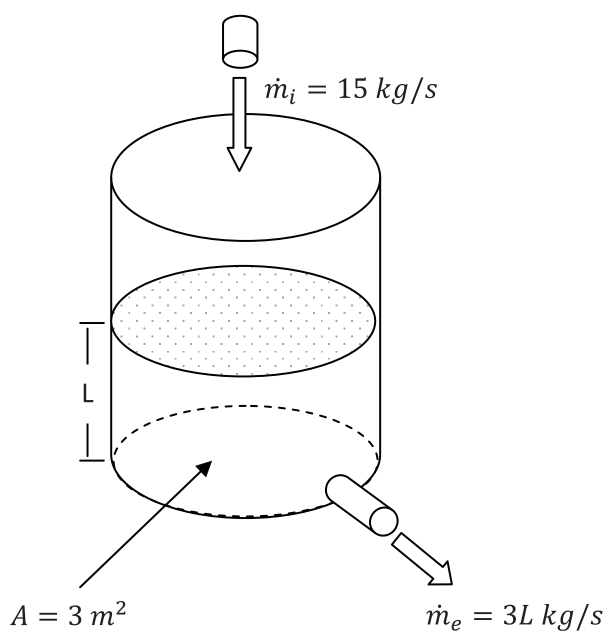
Assignment #4

Assignment date: Wednesday Oct. 7, 2009

Due date: Thursday Oct. 14, 2009

Problem 1

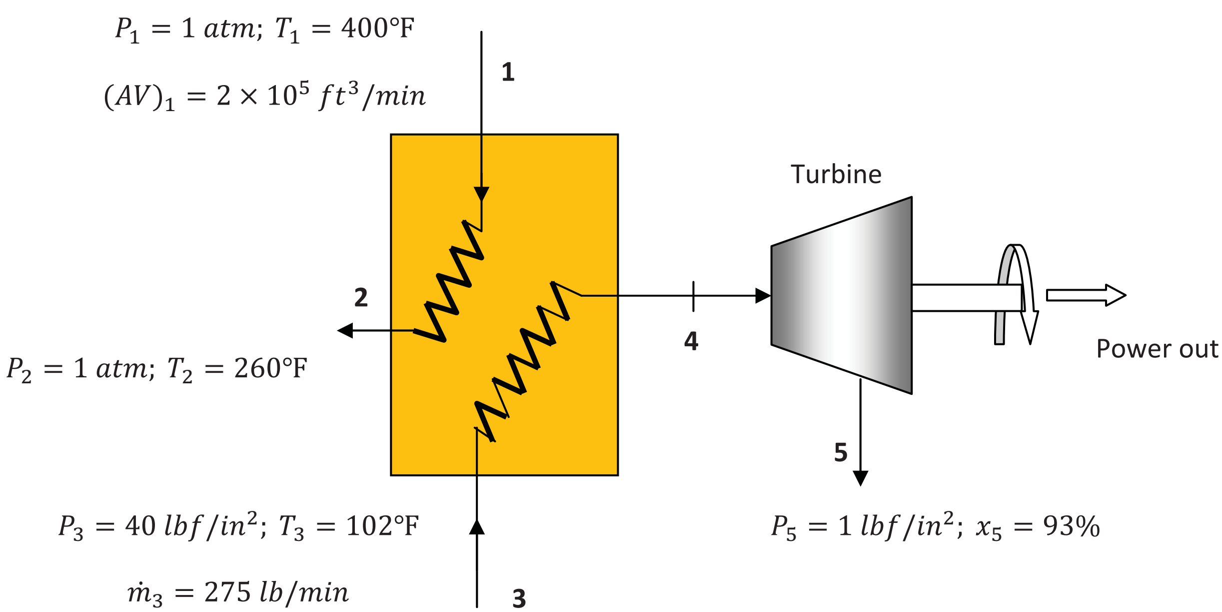
Water flows into the top of an open barrel at a constant mass flow rate of $15\ kg/s$. Water exits through a pipe near the base with a mass flow rate proportional to the height of liquid inside: $\dot{m}_e = 3L$, where L is the instantaneous liquid height, in m. The area of the base is $3\ m^2$, and the density of water is $1000\ kg/m^3$. If the barrel is initially empty, find and plot the variation of liquid height with time and comment on the result.



Problem 2

An industrial process discharges $2 \times 10^5\ ft^3/min$ of gaseous combustion products at 400 °F, 1 atm. As shown in the figure, a proposed system for utilizing the combustion products combines a heat-recovery steam generator with a turbine. At steady state, combustion products exit the steam generator at 260 °F, 1 atm and a separate stream of water enters at $40\ lbf/in^2$, 102°F with a mass flow rate of 275 lb/min. At the exit of the turbine, the pressure is $1\ lbf/in^2$ and the quality is 93%. Heat transfer from the outer surfaces of the steam generator and turbine can be ignored, as can the changes in kinetic and potential energies of the flowing streams. There is no significant pressure drop for the water flowing through the steam generator. The combustion products can be modeled as air as an ideal gas.

a) Determine the power developed by the turbine, in Btu/min.
b) Determine the turbine inlet temperature, in °F.
c) Evaluating the power developed at \$0.08 $per\ kW.h$, which is a typical rate for electricity, determine the value of the power, in $\$/year$, for 8000 hours of operation annually.



Problem 1:

Known:

$$Barrel\ dimensions$$

$$Inlet\ masss\ flow\ rate\ is\ constant$$

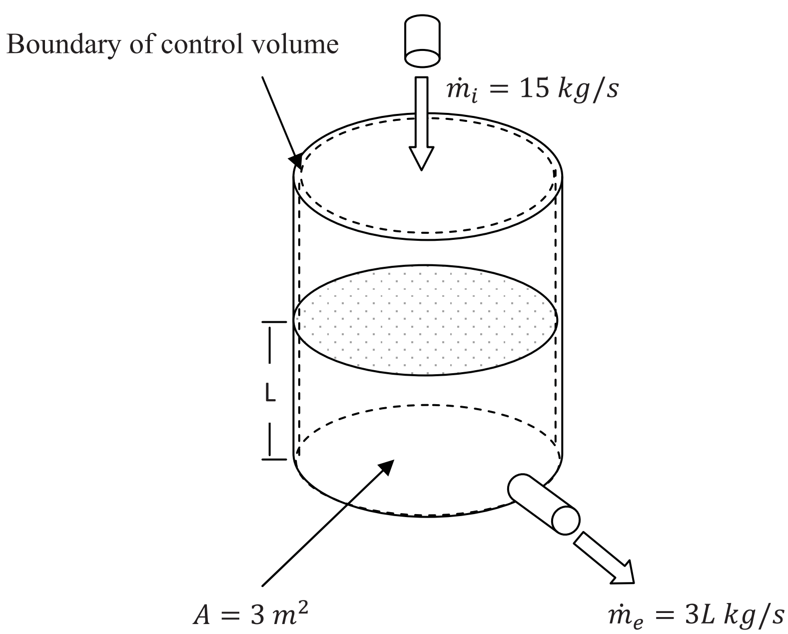
$$Outlet\ mass\ flow\ rate\ is\ proportional\ to\ the\ height\ of\ liquid\ in\ the\ barrel$$

Find:

- Plot of liquid height variation with time and comment.

Assumptions:

- The water density is constant.



Analysis:

Conservation of mass in the control volume:

$$\frac{dm_{CV}}{dt} = \dot{m}_i - \dot{m}_e$$

The mass of water contained within the barrel at time t is given by:

$$m_{CV}(t) = \rho A L(t)$$

where ρ is density, A is the area of the base, and $L(t)$ is the instantaneous liquid height. Substituting this into the mass rate balance together with the given mass flow rates:

$$\frac{d(\rho A L)}{dt} = 15 - 3L(t)$$

Since density and area are constant, this equation can be written as:

$$\frac{dL(t)}{dt} + \left(\frac{3}{\rho A}\right) L = \frac{15}{\rho A}$$

which is a first-order, ordinary differential equation with constant coefficients. The solution is:

$$L(t) = 5 + Cexp\left(-\frac{3t}{\rho A}\right)$$

where C is a constant of integration. The solution can be verified by substituting into the differential equation.

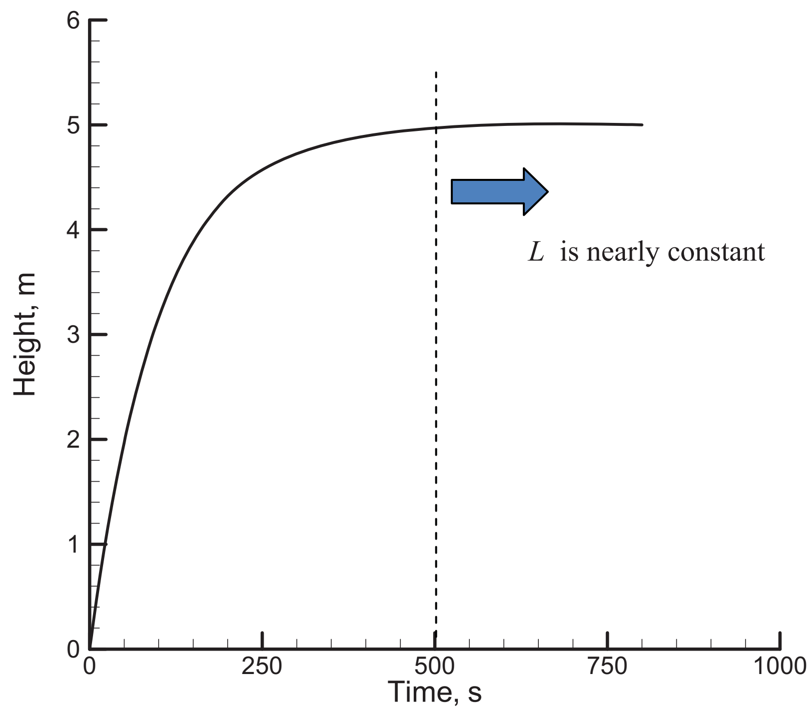
To evaluate C, use the initial condition: $at\ t = 0, L = 0$. Thus, $C = -5$ and the solution can be written as:

$$L(t) = 5\left[1 - exp\left(-\frac{3t}{\rho A}\right)\right]$$

Substituting $\rho = 1000\ kg/m^3$ and $A = 0.3\ m^2$ results in:

$$L(t) = 5[1 - exp(-0.01t)]$$

This relation can be plotted by hand or using appropriate software. The result is:



From the graph, we see that initially the liquid height increases rapidly and then levels out. After about $500\ s$, the height stays nearly constant with time. At this point, the rate of water flow into the barrel nearly equals the rate of flow out of the barrel. From the graph, the limiting value of L is $5\ m$. Which also can be verified by taking the limit of the analytical solution as $t \rightarrow \infty$.

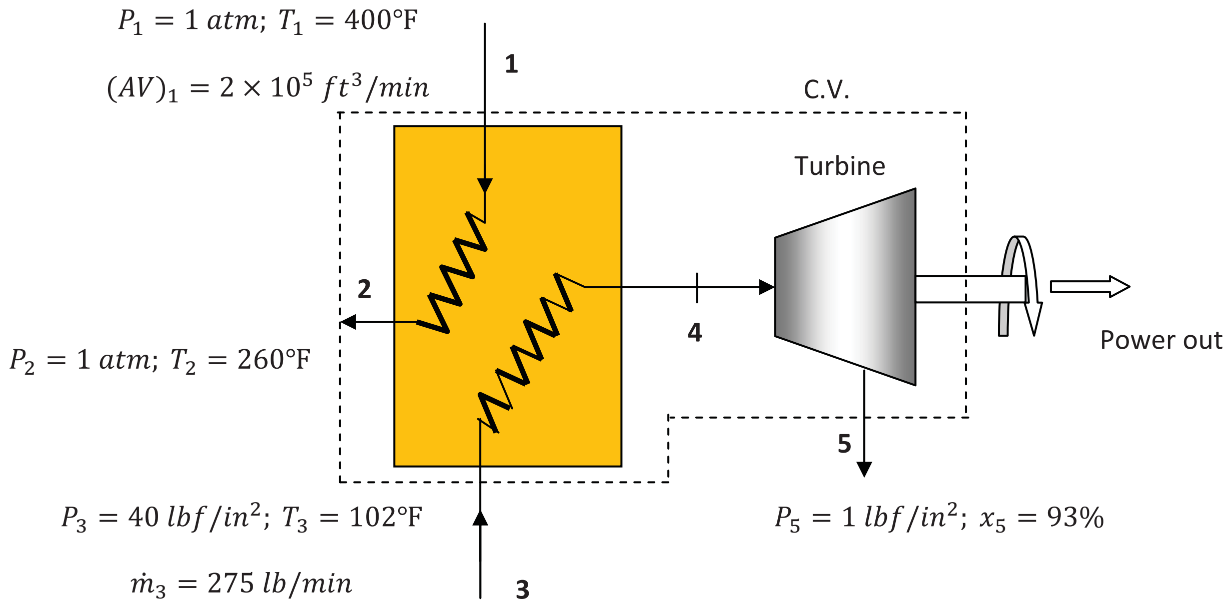
Problem 2:

Known:

$$Steady\ state\ operating\ data\ are\ provided\ for\ the\ system$$

Find:

- Power developed by the turbine.
- Turbine inlet temperature.
- Evaluating annual value of the power developed.



Assumptions:

- The control volume shown in dashed line is at steady state.
- Heat transfer is negligible.
- Changes in kinetic and potential energy can be ignored.
- There is no pressure drop for water flowing through the steam generator.
- The combustion products are modeled as air as an ideal gas.

Analysis:

(a) The power developed by the turbine is determined from a control volume enclosing both steam generator and the turbine. Since the gas and water streams do not mix, mass rate balances for each of streams can be written:

$$\dot{m}_1 = \dot{m}_2 \qquad \dot{m}_3 = \dot{m}_5$$

The steady state form of the energy rate balance is:

$$0 = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{m}_3\left(h_3 + \frac{V_3^2}{2} + gz_3\right) - \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}_5\left(h_5 + \frac{V_5^2}{2} + gz_5\right)$$

Terms containing kinetic and potential energies, $\frac{V_i^2}{2} + gz_i$, drops out by assumptions. Q_{CV} is also zero since there is no heat transfer. With these simplifications, together with the above mass flow rate relations, the energy rate balance becomes:

$$\dot{W}_{CV} = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_5)$$

The mass flow rate $\dot{m}_1$ can be evaluated with the given data at inlet 1 and the ideal gas equation of state:

$$\dot{m}_1 = \frac{(AV)_1 P_1}{(\bar{R}/M)T_1}$$

Using Table A-1E, $M = 28.97\ lbm/lbmol$:

$$\dot{m}_1 = \frac{2\times 10^5 \left[\frac{ft^3}{min}\right] \times 14.7 \left[\frac{lbf}{in^2}\right]}{\dfrac{1545\left[\frac{ft.lbf}{lbmol.°R}\right]}{28.97\left[\frac{lbm}{lbmol}\right]} \times 860[°R]}\left(\frac{144\ [in^2]}{1\ [ft^2]}\right) = 9230.6\ \left[\frac{lb}{min}\right]$$

The specific enthalpies h_1 and h_2 can be found from Table A-17E: At $860°R$, $h_1 = 206.46\ Btu/lbm$ and at $720°R, h_2 = 172.39\ Btu/lbm$. At state 3, water is a liquid. Since at $T_3 = 102,$°F; $P_3 = 1\ lbf/in^2$:

$$h_3 = h_{f@102°\mathrm{F}} + v_{f@102°\mathrm{F}}(P_3 - P_{sat@102°\mathrm{F}})$$

$$h_3 = 68.03\left[\frac{Btu}{lbm}\right] + 0.01613\left[\frac{ft^3}{lbm}\right] \times \left(40\left[\frac{lbf}{in^2}\right] - 1\left[\frac{lbf}{in^2}\right]\right)/5.4\left[\frac{Btu}{\frac{lbf}{in^2}ft^3}\right]$$

$$\rightarrow h_3 = 68.16\left[\frac{Btu}{lbm}\right]$$

State 5 is a two-phase liquid-vapour mixture. With data from Table A-5E, and given quality:

$$h_5 = h_{f@1\,psi} + x_5 h_{fg@1\,psi}$$

$$\rightarrow h_5 = 69.7\left[\frac{Btu}{lbm}\right] + 0.93 \times 1035.7\left[\frac{Btu}{lbm}\right] = 1033.2\left[\frac{Btu}{lbm}\right]$$

Substituting values into the expression for $\dot{W}_{CV}$:

$$\dot{W}_{CV} = 9230.6\ \left[\frac{lb}{min}\right] \times \left(206.46\left[\frac{Btu}{lbm}\right] - 172.39\left[\frac{Btu}{lbm}\right]\right) + 275\ \left[\frac{lb}{min}\right] \times \left(68.16\left[\frac{Btu}{lbm}\right] - 1033.2\left[\frac{Btu}{lbm}\right]\right) = 49610\left[\frac{Btu}{min}\right]$$

(b) To determine T_4, it is necessary to fix the state at 4. This requires two independent property values. With forth assumption, one of the properties is pressure, $P_4 = 40\ lbf/in^2$. The other is specific enthalpy h_4, which can be found from an energy rate balance for a control volume enclosing just the steam generator. Mass rate balances for each of the two streams give $\dot{m}_1 = \dot{m}_2\ and\ \dot{m}_3 = \dot{m}_4$. With the third assumption and these mass flow rate relations, the steady state form of energy rate balance reduces to:

$$0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

Solving for h_4:

$$h_4 = \frac{\dot{m}_1}{\dot{m}_3}(h_1 - h_2) + h_3$$

$$h_4 = \frac{9230.6\ \left[\frac{lb}{min}\right]}{275\ \left[\frac{lb}{min}\right]}\left(206.46\left[\frac{Btu}{lbm}\right] - 172.39\left[\frac{Btu}{lbm}\right]\right) + 68.16\left[\frac{Btu}{lbm}\right]$$

$$\rightarrow h_4 = 1213.6\left[\frac{Btu}{lbm}\right] > h_{g\ @\ 40\ psi} = 1169.8\left[\frac{Btu}{lbm}\right]$$

Since state 4 is superheated vapour, Table A-6E should be used. Interpolating at $P_4 = 40\ psi$, we get $T_4 = 354$ °F.

(c) Using the result of part (a), together with given economic data and appropriate conversion factors, the value of the power developed for 8000 hours of operation annually is:

$$Annual\ val. = \left(49610\left[\frac{Btu}{min}\right] \times \frac{60\ [min]}{1\ [hr]} \times \frac{1\ [kW]}{3413\ \left[\frac{Btu}{hr}\right]}\right)\left(8000\left[\frac{hr}{yr}\right]\right)\left(0.08\left[\frac{\$}{kW.hr}\right]\right) = 558{,}000\left[\frac{\$}{yr}\right]$$

Note (1): Alternatively, to determine state 4, a control volume enclosing just the turbine can be considered.