ENSC 388

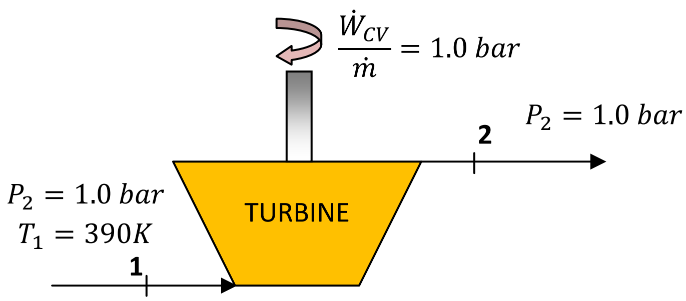
Assignment #6

Assignment date: Wednesday Oct. 21, 2009

Due date: Wednesday Oct. 28, 2009

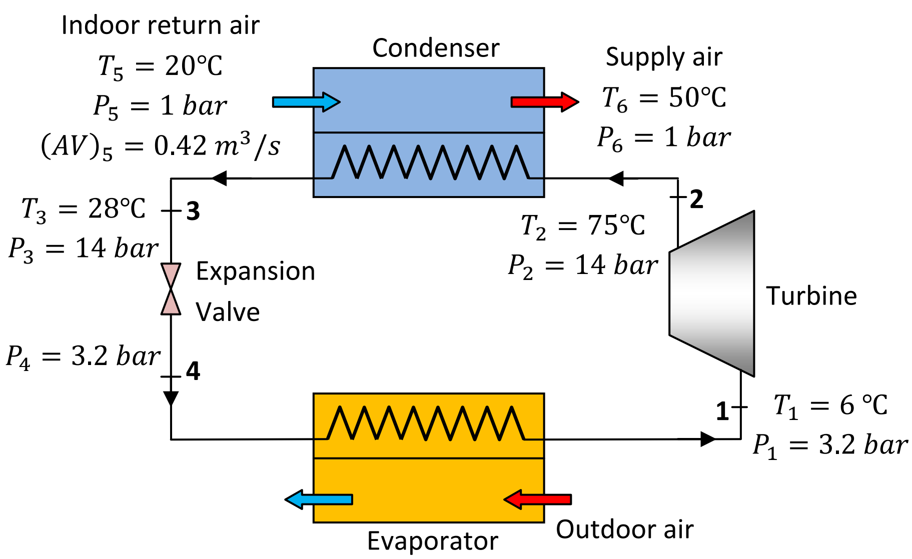
Problem 1

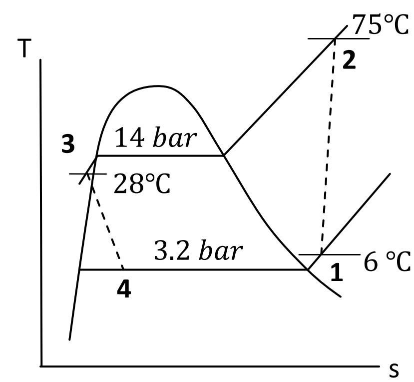
A turbine operating at steady state receives air at a pressure of $P_1 = 3.0\ bar$ and a temperature of $T_1 = 390K$. Air exits the turbine at a pressure of $P_2 = 1.0\ bar$. The work developed is measured as $74\ kJ$ per kg of air flowing through the turbine. The turbine operates adiabatically, and changes in kinetic and potential energy between inlet and exit can be neglected. Using the ideal gas model for air, determine the turbine efficiency.



Problem 2

Components of a heat pump for supplying heated air to a dwelling are shown in the schematic below. At steady state, Refrigerant 134a enters the compressor at 6 ℃, 3.2 bar and is compressed adiabatically to 75 ℃, 14 bar. From the compressor, the refrigerant passes through the condenser, where it condenses to liquid at 28 ℃, 14 bar. The refrigerant then expands through a throttling valve to 3.2 bar. The states of the refrigerant are shown on the accompanying T-s diagram. Return air from the dwelling enters the condenser at $20\ ℃$, $1\ bar$ with a volumetric flow rate of $0.42\ m^3/s$ and exits at $50\ ℃$ with a negligible change in pressure. Using the ideal gas model for the air and neglecting kinetic and potential energy effects, (a) determine the rates of entropy production, in kW/K, for control volumes enclosing the condenser, compressor, and expansion valve, respectively. (b) Discuss the sources of irreversibility in the components considered in part (a).





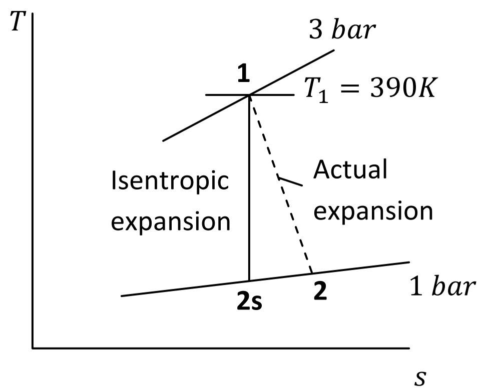
Problem 1:

Known:

$$States\ information\ and\ the\ processes.$$

Find:

- Turbine efficiency.



Assumptions:

- Problem is steady state.
- The expansion is adiabatic and the changes in kinetic and potential energy between inlet and exit can be neglected.
- The air is modeled as an ideal gas.

Analysis:

Isentropic turbine efficiency can be obtained from

$$\eta_t = \frac{\frac{\dot{W}_{CV}}{\dot{m}}}{\left(\frac{\dot{W}_{CV}}{\dot{m}}\right)_s}$$

The numerator of the efficiency is known. The denominator is evaluated as follows.

The work developed in an isentropic expansion from the given inlet state to the specified exit pressure is

$$\left(\frac{\dot{W}_{CV}}{\dot{m}}\right)_s = h_1 - h_{2s}$$

From Table A-17, at $390K$, $h_1 = 390.88\ kJ/kg$. To determine h_{2s}, applying an exact solution leads to

$$P_r(T_{2s}) = \left(\frac{P_2}{P_1}\right) P_r(T_1)$$

With $P_1 = 3\ bar$, $P_2 = 1\ bar$ and $P_r(T_1) = 3.481$ form Table A-17 at $390K$

$$P_r(T_{2s}) = \left(\frac{1}{3}\right) \times 3.481 = 1.1603$$

Interpolating in Table A-17 gives $h_{2s} = 285.27\ kJ/kg$. Thus

$$\left(\frac{\dot{W}_{CV}}{\dot{m}}\right)_s = 390.88 \left[\frac{kJ}{kg}\right] - 285.27 \left[\frac{kJ}{kg}\right] = 105.6 \left[\frac{kJ}{kg}\right]$$

Hence isentropic turbine efficiency will be

$$\eta_t = \frac{74 \left[\frac{kJ}{kg}\right]}{105.6 \left[\frac{kJ}{kg}\right]} = 0.7\ or\ 70\%$$

Problem 2:

Known:

$$States\ information\ and\ the\ processes.$$

Find:

- The entropy production rates for control volumes enclosing the condenser, compressor, and expansion valve, respectively and discuss the sources of irreversibility in these components.

Assumptions:

- Each component is analyzed as a control volume at steady state.
- The compressor operates adiabatically, and the expansion across the valve is a throttling process.
- For the control volume enclosing the condenser, $\dot{W}_{CV} = 0$ and $\dot{Q}_{CV} = 0$.
- Kinetic and potential energy effects can be neglected.
- The air is modeled as an ideal gas with constant $c_p = 1.005\ kJ/kg.K$.

Analysis:

(a) Lets us begin by obtaining property data at each of the principal refrigerant states located on the *T-s* diagram. At the inlet to the compressor, the refrigerant is a superheated vapour at 6 ℃, 3.5 bar, so from Table A-13, $s_1 = 0.9415\ kJ/kg.K$. Similarly, at state 2, the refrigerant is superheated vapour at 75 ℃, $14\ bar$, so interpolating in Table A-13 gives $s_2 = 0.9561\ kJ/kg.K$ and $h_2 = 291.3\ kJ/kg$. State 3 is compressed liquid at 28 ℃, $14\ bar$. From Table A-11, $s_3 \approx s_{f@28\,℃} = 0.33846\ kJ/kg.K$ and $h_3 \approx h_{f@28\,℃} = 90.69\ kJ/kg$. The expansion through the valve is a throttling process, thus $h_4 = h_3$. Using data from Table A-11, the quality at state 4 is

$$x_4 = \frac{h_4 - h_{f@3.2bar}}{h_{fg@3.2bar}} = \frac{90.69\ \left[\frac{kJ}{kg}\right] - 55.16\left[\frac{kJ}{kg}\right]}{196.71\left[\frac{kJ}{kg}\right]} = 0.18$$

And the specific entropy is

$$s_4 = s_{f@3.2bar} + x_4 s_{fg@3.2bar} = 0.21637\left[\frac{kJ}{kg.K}\right] + 0.18 \times 0.71369\left[\frac{kJ}{kg.K}\right]$$

$$\rightarrow s_4 = 0.3448\left[\frac{kJ}{kg.K}\right]$$

Condenser:

Consider the control volume enclosing the condenser. With the first and third assumptions, the entropy rate balance reduces to

$$0 = \dot{m}_{ref}(s_2 - s_3) + \dot{m}_{air}(s_5 - s_6) + \dot{s}_{gen,cond}$$

To evaluate $\dot{s}_{gen}$ requires the two mass flow rates, $\dot{m}_{ref}$ and $\dot{m}_{air}$, and the change in specific entropy for the air. These are obtained next.
Evaluating the mass flow rate of air using the ideal gas model

$$\dot{m}_{air} = (AV)_5 \underbrace{\frac{P_5}{RT_5}}_{\rho} = 0.42\left[\frac{m^3}{s}\right] \times \frac{100[kPa]}{0.286\left[\frac{kJ}{kg.K}\right] \times 293\,[K]} = 0.5\left[\frac{kg}{s}\right]$$

The refrigerant mass flow rate is determined using an energy balance for the control volume enclosing the condenser together with the 1st, 3rd and 4th assumptions to obtain

$$\dot{m}_{ref} = \frac{\dot{m}_{air}(h_6 - h_5)}{h_2 - h_3}$$

With the 5th assumption, $h_6 - h_5 = c_p(T_6 - T_5)$. Inserting values

$$\dot{m}_{ref} = \frac{0.5\left[\frac{kg}{s}\right] \times 1.005\left[\frac{kJ}{kg.K}\right] \times (323[K] - 293[K])}{291.3\left[\frac{kJ}{kg}\right] - 90.69\left[\frac{kJ}{kg}\right]} = 0.075\ \left[\frac{kg}{s}\right]$$

The change in specific entropy of the air is

$$s_6 - s_5 = c_p ln\frac{T_6}{T_5} - Rln\frac{P_6}{P_5} = 1.005\left[\frac{kJ}{kg.K}\right] \times ln\frac{323[K]}{293[K]} - R\underbrace{ln\frac{1[bar]}{1[bar]}}_{0}$$

$$\rightarrow s_6 - s_5 = 0.098\left[\frac{kJ}{kg.K}\right]$$

Finally, solving the entropy balance for $\dot{s}_{gen}$ and inserting values

$$\dot{s}_{gen} = \dot{m}_{ref}(s_3 - s_2) + \dot{m}_{air}(s_6 - s_5)$$

$$\dot{s}_{gen,cond} = 0.075\ \left[\frac{kg}{s}\right] \times \left(0.33846\left[\frac{kJ}{kg.K}\right] - 0.9561\left[\frac{kJ}{kg.K}\right]\right) + 0.5\left[\frac{kg}{s}\right] \times 0.098\left[\frac{kJ}{kg.K}\right] = 2.677 \times 10^{-3}\left[\frac{kW}{K}\right]$$

Compressor:

For the control volume enclosing the compressor, the entropy rate balance reduces with the first and third assumptions to

$$0 = \dot{m}_{ref}(s_1 - s_2) + \dot{s}_{gen,comp}$$

or

$$\dot{s}_{gen,comp} = \dot{m}_{ref}(s_2 - s_1) = 0.075\ \left[\frac{kg}{s}\right] \times \left(0.9561\left[\frac{kJ}{kg.K}\right] - 0.9415\left[\frac{kJ}{kg.K}\right]\right) = 1.095 \times 10^{-3}\left[\frac{kW}{K}\right]$$

Valve:

Finally, for the control volume enclosing the throttling valve, the entropy rate balance reduces to

$$0 = \dot{m}_{ref}(s_3 - s_4) + \dot{s}_{gen,val}$$

Solving for $\dot{s}_{gen,val}$ and inserting values

$$\dot{s}_{gen,val} = \dot{m}_{ref}(s_4 - s_3) = 0.075\ \left[\frac{kg}{s}\right] \times \left(0.3448 \left[\frac{kJ}{kg.K}\right] - 0.33846\ \left[\frac{kJ}{kg.K}\right]\right)$$

$$= 4.755 \times 10^{-4} \left[\frac{kW}{K}\right]$$

(b) The following table summarize, in rank order, the calculated entropy production rates

Component	$\dot{s}_{gen} \left[\frac{kW}{K}\right]$
compressor	1.095×10^{-3}
valve	4.755×10^{-4}
condenser	2.677×10^{-3}

Entropy production in the compressor is due to fluid friction, mechanical friction of the moving parts, and internal heat transfer. For the valve, the irreversibility is primarily due to fluid friction accompanying the expansion across the valve. The principal source of irreversibility in the condenser is the temperature difference between the air and refrigerant streams. In this problem, there are no pressure drops for either stream passing through the condenser, but slight pressure drops due to fluid friction would normally contribute to the irreversibility of condensers. The evaporator lightly has not analyzed in this solution.

Note (1): Since the temperature difference is large in problem one, applying approximate solution may leads to considerable error. Try to apply the approximate solution and see how different the result is.

Note (2): Due to relatively small temperature change of the air, the specific heat c_p can be taken constant at the average of the inlet and exit air temperature.

Note (3): Temperature in K are used to evaluate $\dot{m}_{ref}$, but since a temperature **difference** is involved the same result would be obtained if temperature in ℃ were used. Temperatures in K must be used when a temperature **ratio** is involved.