ENSC 388

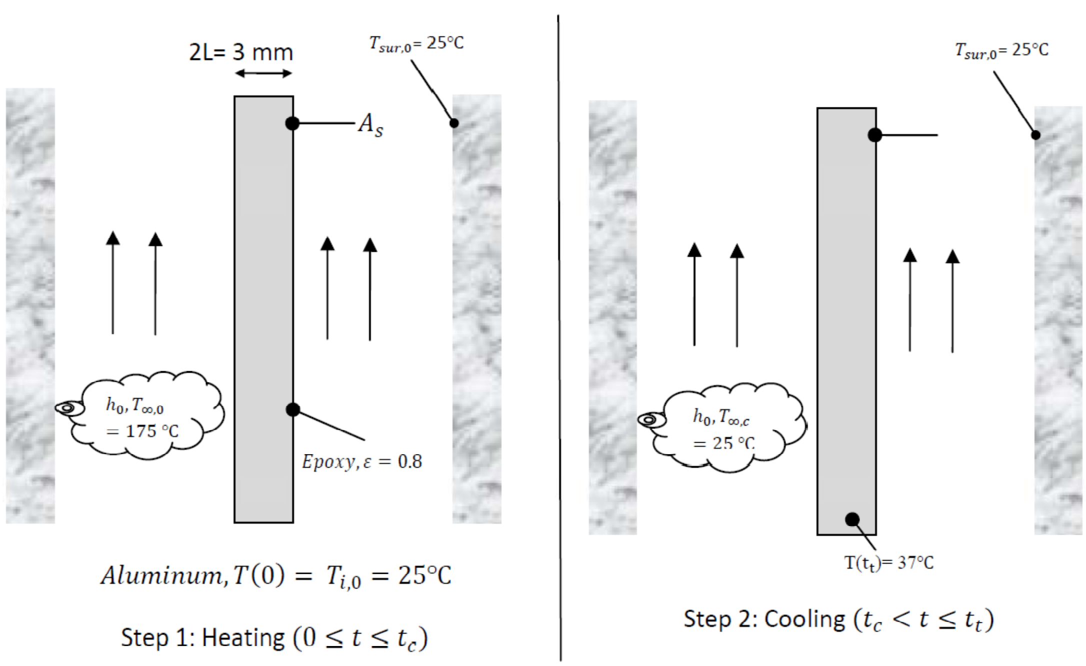
Assignment #8

Assignment date: Wednesday Nov. 11, 2009

Due date: Wednesday Nov. 18, 2009

Problem 1

A 3-*mm*-thick panel of aluminum alloy (k = *177 W/m·K*, c = *875 J/kg·K*, and ρ = *2770 kg/m³*) is finished on both sides with an epoxy coating that must be cured at or above T_c = *150°C* for at least 5 min. The production line for the curing operation involves two steps: (1) heating in a large oven with air at $T_{\infty,o}$= *175°C* and a convection coefficient of h_o= *40 W/m²·K*, and (2) cooling in a large chamber with air a $T_{\infty,c}$= *25°C* and a convection coefficient of h_c= *10 W/m²·K*. The heating portion of the process is conducted over a time interval t_e, which exceeds the time t_c required to reach *150°C* by 5 min (t_e = t_c + 300 s). The coating has an emissivity of $\varepsilon = 0.8$, and the temperatures of the oven and chamber walls are 175°C and *25°C*, respectively. If the panel is placed in the oven at an initial temperature of 25°C and removed from the chamber at a *safe-to-touch* temperature of *37°C*, what is the total elapsed time for the two-step curing operation?



Problem 2

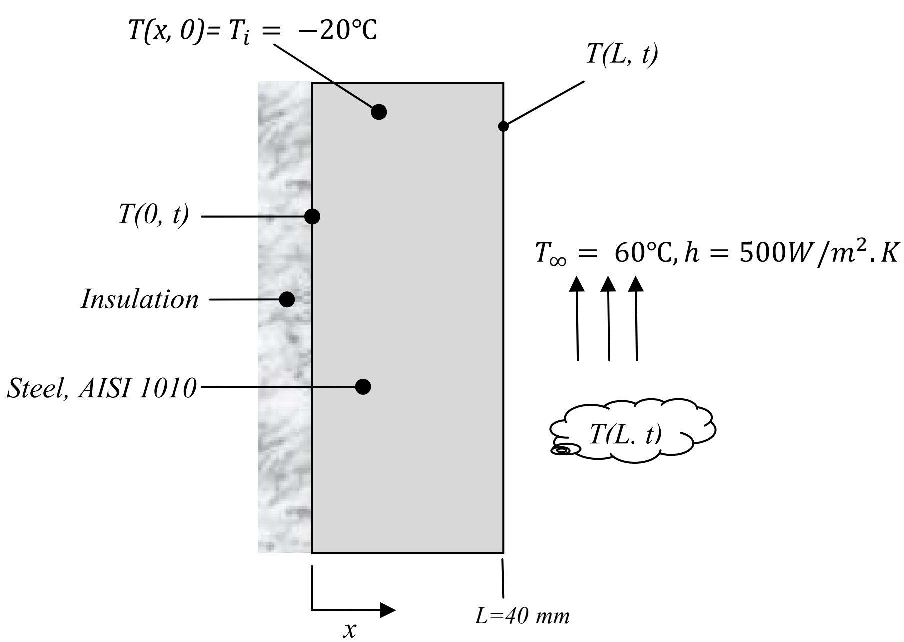
Consider a steel pipeline (AISI 1010) that is 1 *m* in diameter and has a wall thickness of 40 *mm*. The pipe is heavily insulated on the outside, and before the initiation of flow, the walls of the pipe are at a uniform temperature of -*20°C*. With the initiation of flow, hot oil at *60°C* is pumped through the pipe, creating a convective surface condition corresponding to $h = 500\ W/m^2{\cdot}K$ at the inner surface of the pipe.

1. What are the appropriate Biot and Fourier numbers 8 min after the initiation of flow?

2. At $t = 8$ min, what is the temperature of the exterior pipe surface covered by the insulation?

3. What is the heat flux q'' (W/m^2) to the pipe from the oil at $t = 8$ min?

4. How much energy per meter of pipe length has been transferred from the oil to the pipe at $t = 8$ min?



Problem 1:

Known:

Operating conditions for a two-step heating/cooling process in which a coated aluminum panel is maintained at or above a temperature of l50°C for at least 5 min.

Find:

- Total time t_t required for the two-step process.

Assumptions:

1. Panel temperature is uniform at any instant.
2. Thermal resistance of epoxy is negligible.
3. Constant properties.

Analysis:

To assess the validity of the lumped capacitance approximation, we begin by calculating Biot numbers for the heating and cooling processes.

$$Bi_h = \frac{h_o L}{k} = \frac{(40\ W/m^2.K)(0.0015m)}{177\ W/m.K} = 3.4 \times 10^{-4}$$

$$Bi_c = \frac{h_c L}{k} = \frac{(10\ W/m^2.K)(0.0015m)}{177\ W/m.K} = 8.5 \times 10^{-5}$$

Hence the lumped capacitance approximation is excellent.
To determine whether radiation exchange between the panel and its surroundings should be considered, the radiation heat transfer coefficient is determined from:

$$h_r = \varepsilon\sigma\left(T_c + T_{sur,o}\right)(T_c^2 + T_{sur,o}^2)$$

A representative value of h, for the heating process is associated with the cure condition, in which case

$$h_r = \varepsilon\sigma\left(T_c + T_{sur,o}\right)\left(T_c^2 + T_{sur,o}^2\right)$$

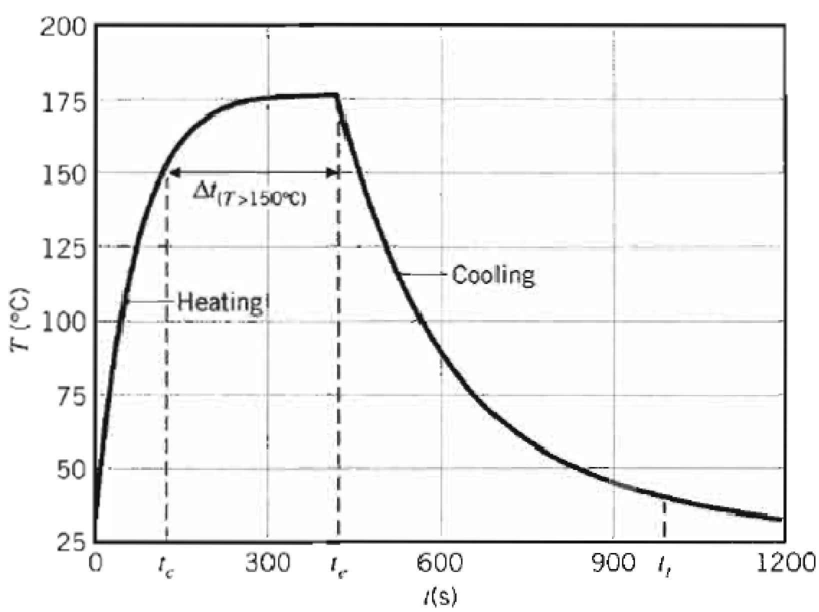
$$= 0.8 \times 5.67 \times 10^{-8}\ W/m^2.K^4 (423 + 448)K(423^2 + 448^2)K^2 = 15\ W/m^2.K$$

Using T_c = 150°C with $T_{sur,c}$ = 25°C for the cooling process, we also obtain $h_{r,c}$ = 5.1 $W/m^2.K$. Since the values of $h_{r,o}$, and $h_{r,c}$ are comparable to those of h_o and h_c, respectively, radiation effects must be considered.

With $V = 2LA_s$, and $A_{s,c} = A_{s,r} = 2A_s$, applying conservation of energy:

$$\int_{T_i}^{T} dT = T(t) - T_i = -\frac{1}{\rho c L}\int_0^t [h(T - T_\infty) + \varepsilon\sigma(T^4 + T_{sur}^4)]\, dt$$

Selecting a suitable time increment Δt, the right-hand side of this equation may be evaluated numerically to obtain the panel temperature at t= Δt, 2Δt, 3 Δt, and so on. At each new step of the calculation, the value of T computed from the previous time step is used in the integrand, Selecting Δt = 10 s, calculations for the heating process are extended to $t_e = t_c + 300$ s, which is 5 min beyond the time required for the panel to reach T_c = 150°C. At t_e the cooling process is initiated and continued until the panel temperature reaches 37°C at $t = t_t$. The integration was performed using a fourth-order Runge-Kutta scheme, and results of the calculations are plotted as follows:



The total time for the two-·step process is

$$t_t = 989\ s$$

with intermediate times of t_c= 124 s and t_e = 424 s.

Problem 2:

Known:

Wall subjected to sudden change in convective surface condition.

Find:

1. Biot and Fourier numbers after 8 min.
2. Temperature of exterior pipe surface after 8 min, l
3. Heat flux to the wall at 8 min. l
4. Energy transferred to pipe per unit length after 8 min.

Assumptions:

1. Pipe wall can be approximated as plane wall, since thickness is much less than diameter.
2. Constant properties.
3. Outer surface of pipe is adiabatic.

Properties:

Table A. 24, steel type AISI 1010 [T= (-20+60) °C/2≈ 300k]: ρ=7823 kg/m³, c= 434 J/kg.K, k= 63.9 W/m.K, α= $18.8 \times 10^{-6}\ m^2/s$.

Analysis:

1. At t = 8 min, the Biot and Fourier numbers are respectively, with $L_c = L$. Hence

$$Bi = \frac{hL}{k} = \frac{500\ W/m^2.K \times 0.04m}{63.9\ W/m.K} = 0.313$$

$$Fo = \frac{\alpha t}{L^2} = \frac{18.8 \times 10^{-6}\ m^2/s \times 8\ min \times 60\ s/m}{63.9\ W/m.K} = 5.64$$

2. With Bi = 0.313, use of the lumped capacitance method is inappropriate. However, since Fo > 0.2 and transient conditions in the insulated pipe wall of thickness L correspond to those in a plane wall of thickness 2L experiencing the same surface condition, the desired results may be obtained from the one-term approximation for a plane wall. The midplane temperature can be determined from:

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\xi_1^2 \, F_o\right)$$

where, with Bi = 0.1313, C_1 = 1.047 and ξ_1 = 0.531 rad from Table 11.2. With $Fo = 5.64$,

$$\theta_o^* = 1.047 \exp[-(0.531\, rad)^2 \times 5.64] = 0.214$$

Hence after 8 min, the temperature of the exterior pipe surface, which corresponds to the midplane temperature of a plane wall, is

$$T(0, 8 \text{ min}) = T_\infty + \theta_o^*(T_i - T_\infty) = 60℃ + 0.214\,(-20 - 60)℃ = 42.9℃$$

3. Heat transfer to the inner surface at $x = L$ is by convection, and at any time t the heat flux may be obtained from Newton`s law of cooling. Hence at t = 480 s,

$$q_x^{"}(L, 480\, s) = q_L^{"} = h[T(L, 480\, s) - T_\infty]$$

Using the one-term approximation for the surface temperature, Equation:

$$\theta^* = \theta_o^* \cos\left(-\xi_1 \;\; x^*\right)$$

with x^* = 1 has the form

$$\theta^* = \theta_o^* \cos\left(-\xi_1\right)$$

$$T(L, t) = T_\infty + (T_i - T_\infty)\theta_o^* \cos\left(-\xi_1\right)$$

$$T(L, 8 \text{ min}) = 60℃ + (-20 - 60) \times 0.214 \times \cos\left(0.531\, rad\right) = 45.2℃$$

The heat flux at t = 8 min is then

$$q_L^{"} = 500 \frac{W}{m^2}.K\;(45.2 - 60)℃ = -7400\; W/m^2$$

4. The energy transfer to the pipe wall over the 8-min interval may be obtained from following Equations

$$\frac{Q}{Q_o} = 1 - \frac{\sin\left(\xi_1\right)}{\xi_1} \theta_o^*$$

$$\frac{Q}{Q_o} = 1 - \frac{\sin\left(0.531\, rad\right)}{0.531\, rad} \times 0.214 = 0.80$$

it follows that

$$Q = 0.8\,\rho c V(T_i - T_\infty)$$

or with a volume per unit pipe length of V' = πDL,

$$\acute{Q} = 0.8\,\rho c \pi D L(T_i - T_\infty)$$

$$\acute{Q} = 0.8 \times 7823\frac{kg}{m^3} \times 434\frac{J}{kg}.K \times \pi \times 1m \times 0.04m(-20-60)℃$$

$$\acute{Q} = -2.73 \times 10^7 J/m$$