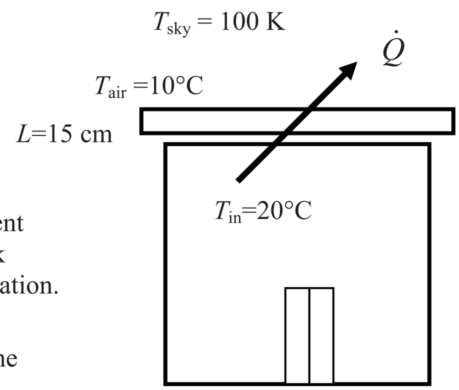
10-30 The roof of a house with a gas furnace consists of a concrete that is losing heat to the outdoors by radiation and convection. The rate of heat transfer through the roof and the money lost through the roof that night during a 14 hour period are to be determined.



Assumptions **1** Steady operating conditions exist. **2** The emissivity and thermal conductivity of the roof are constant.

Properties The thermal conductivity of the concrete is given to be k = 2 W/m·°C. The emissivity of both surfaces of the roof is given to be 0.9.

Analysis When the surrounding surface temperature is different than the ambient temperature, the thermal resistances network approach becomes cumbersome in problems that involve radiation. Therefore, we will use a different but intuitive approach.

In steady operation, heat transfer from the room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), that must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{room to roof, conv+rad}} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

Taking the inner and outer surface temperatures of the roof to be $T_{s,in}$ and $T_{s,out}$, respectively, the quantities above can be expressed as

$$\dot{Q}_{\text{room to roof, conv+rad}} = h_i A(T_{room} - T_{s,in}) + \varepsilon A\sigma(T_{room}{}^4 - T_{s,in}{}^4) = (5\ \text{W/m}^2\cdot°\text{C})(300\ \text{m}^2)(20 - T_{s,in})°\text{C}$$
$$+ (0.9)(300\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(20+273\ \text{K})^4 - (T_{s,in}+273\ \text{K})^4\right]$$

$$\dot{Q}_{\text{roof, cond}} = kA\frac{T_{s,in} - T_{s,out}}{L} = (2\ \text{W/m}\cdot°\text{C})(300\ \text{m}^2)\frac{T_{s,in} - T_{s,out}}{0.15\ \text{m}}$$

$$\dot{Q}_{\text{roof to surr, conv+rad}} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}{}^4 - T_{surr}{}^4) = (12\ \text{W/m}^2\cdot°\text{C})(300\ \text{m}^2)(T_{s,out} - 10)°\text{C}$$
$$+ (0.9)(300\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(T_{s,out}+273\ \text{K})^4 - (100\ \text{K})^4\right]$$

Solving the equations above simultaneously gives

$$\dot{Q} = \mathbf{37{,}440\ W},\ T_{s,in} = \mathbf{7.3°C},\ \text{and}\ T_{s,\text{out}} = -2.1°\text{C}$$

The total amount of natural gas consumption during a 14-hour period is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{\dot{Q}\Delta t}{0.80} = \frac{(37.440\ \text{kJ/s})(14\times3600\ \text{s})}{0.80}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right) = 22.36\ \text{therms}$$

Finally, the money lost through the roof during that period is

Money lost = (22.36 therms)($1.20 / therm) = **$26.8**