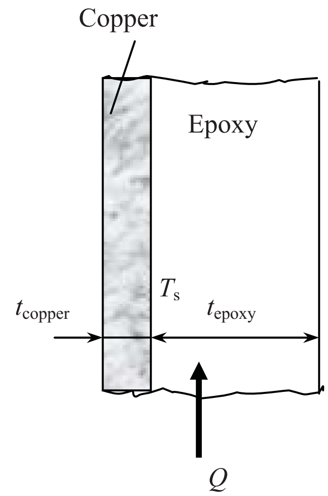
10-38 Heat is to be conducted along a circuit board with a copper layer on one side. The percentages of heat conduction along the copper and epoxy layers as well as the effective thermal conductivity of the board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces is disregarded **3** Thermal conductivities are constant.



Properties The thermal conductivities are given to be k = 386 W/m·°C for copper and 0.26 W/m·°C for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer board can be expressed as

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}}$$

$$= \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right]w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" board of thickness $t = t_{copper} + t_{epoxy}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}(t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff}(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fractions of heat conducted along the copper and epoxy layers as well as the effective thermal conductivity of the board are determined to be

$$(kt)_{copper} = (386\text{ W/m}\cdot\text{°C})(0.0001\text{ m}) = 0.0386\text{ W/°C}$$

$$(kt)_{epoxy} = (0.26\text{ W/m}\cdot\text{°C})(0.0012\text{ m}) = 0.000312\text{ W/°C}$$

$$(kt)_{total} = (kt)_{copper} + (kt)_{epoxy} = 0.0386 + 0.000312 = 0.038912\text{ W/°C}$$

$$f_{epoxy} = \frac{(kt)_{epoxy}}{(kt)_{total}} = \frac{0.000312}{0.038912} = 0.008 = \mathbf{0.8\%}$$

$$f_{copper} = \frac{(kt)_{copper}}{(kt)_{total}} = \frac{0.0386}{0.038912} = 0.992 = \mathbf{99.2\%}$$

and

$$k_{eff} = \frac{(386 \times 0.0001 + 0.26 \times 0.0012)\text{ W/°C}}{(0.0001 + 0.0012)\text{ m}} = \mathbf{29.9\text{ W/m.°C}}$$