10-71 A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be k = 15 W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7$ kJ/kg. The outer surface of the tank is black and thus its emissivity is ε = 1.

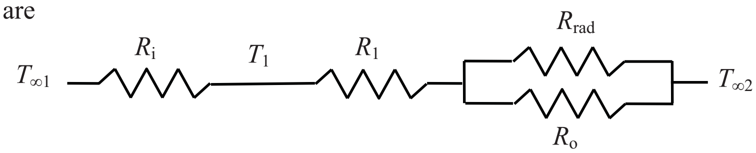
Analysis (*a*) The inner and the outer surface areas of sphere are

$$A_i = \pi D_i^2 = \pi(8\text{ m})^2 = 201.06\text{ m}^2 \qquad A_o = \pi D_o^2 = \pi(8.03\text{ m})^2 = 202.57\text{ m}^2$$

We assume the outer surface temperature T_2 to be 5°C after comparing convection heat transfer coefficients at the inner and the outer surfaces of the tank. With this assumption, the radiation heat transfer coefficient can be determined from

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2^2 + T_{surr}^2)(T_2 + T_{surr}) \\ &= 1(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(273+5\text{ K})^2 + (273+25\text{ K})^2](273+25\text{ K})(273+5\text{ K})] \\ &= 5.424\text{ W/m}^2\text{.K} \end{aligned}$$

The individual thermal resistances are



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(80\text{ W/m}^2\cdot{}^\circ\text{C})(201.06\text{ m}^2)} = 0.000062\ {}^\circ\text{C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(4.015-4.0)\text{ m}}{4\pi(15\text{ W/m}\cdot{}^\circ\text{C})(4.015\text{ m})(4.0\text{ m})} = 0.000005\ {}^\circ\text{C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(10\text{ W/m}^2\cdot{}^\circ\text{C})(202.57\text{ m}^2)} = 0.000494\ {}^\circ\text{C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A} = \frac{1}{(5.424\text{ W/m}^2\cdot{}^\circ\text{C})(202.57\text{ m}^2)} = 0.000910\ {}^\circ\text{C/W}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{conv,o}} + \frac{1}{R_{rad}} = \frac{1}{0.000494} + \frac{1}{0.000910} \longrightarrow R_{eqv} = 0.000320\ {}^\circ\text{C/W}$$

$$R_{total} = R_{conv,i} + R_1 + R_{eqv} = 0.000062 + 0.000005 + 0.000320 = 0.000387\ {}^\circ\text{C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(25-0)^\circ\text{C}}{0.000387\ {}^\circ\text{C/W}} = \mathbf{64{,}600\ W}$$

(*b*) The total amount of heat transfer during a 24-hour period and the amount of ice that will melt during this period are

$$Q = \dot{Q}\Delta t = (64.600\text{ kJ/s})(24\times3600\text{ s}) = 5.581\times10^6\text{ kJ}$$

$$m_{ice} = \frac{Q}{h_{if}} = \frac{5.581\times10^6\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{16{,}730\ kg}$$

Check: The outer surface temperature of the tank is

$$\dot{Q} = h_{conv+rad} A_o (T_{\infty1} - T_s)$$

$$\rightarrow T_s = T_{\infty1} - \frac{\dot{Q}}{h_{conv+rad} A_o} = 25^\circ\text{C} - \frac{64{,}600\text{ W}}{(10+5.424\text{ W/m}^2\cdot{}^\circ\text{C})(202.57\text{ m}^2)} = 4.3^\circ\text{C}$$

which is very close to the assumed temperature of 5°C for the outer surface temperature used in the evaluation of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations.