10-85 A spherical tank filled with liquid nitrogen at 1 atm and -196°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid nitrogen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

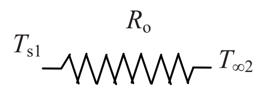
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the nitrogen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid nitrogen at 1 atm are given to be 198 kJ/kg and 810 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (*a*) The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\text{ m})^2 = 28.27\text{ m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(28.27\text{ m}^2)} = 0.00101\,°\text{C/W}$$

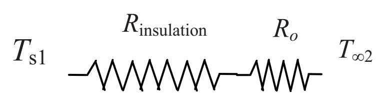


$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-196)]°\text{C}}{0.00101\,°\text{C/W}} = 208{,}910\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{208.910\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{1.055\text{ kg/s}}$$

(*b*) The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\text{ m})^2 = 30.19\text{ m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(30.19\text{ m}^2)} = 0.000946\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\text{ m}}{4\pi(0.035\text{ W/m.°C})(1.55\text{ m})(1.5\text{ m})} = 0.0489\,°\text{C/W}$$

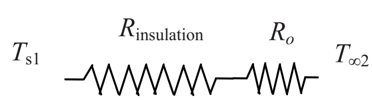
$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{0.0498\,°\text{C/W}} = 4233\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{4.233\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{0.0214\text{ kg/s}}$$

(*c*) The heat transfer rate and the rate of evaporation of the liquid with 2-cm thick layer of superinsulation is

$$A = \pi D^2 = \pi(3.04\text{ m})^2 = 29.03\text{ m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\text{ W/m}^2.°\text{C})(29.03\text{ m}^2)} = 0.000984\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\text{ m}}{4\pi(0.00005\text{ W/m.°C})(1.52\text{ m})(1.5\text{ m})} = 13.96\,°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{13.96\,°\text{C/W}} = 15.11\text{ W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01511\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{0.000076\text{ kg/s}}$$