

Critical Radius of Insulation

10-87C In a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of insulation, but decreases the convection resistance of the surface because of the increase in the outer surface area. Due to these opposite effects, a critical radius of insulation is defined as the outer radius that provides maximum rate of heat transfer. For a cylindrical layer, it is defined as $r_{cr} = k/h$ where k is the thermal conductivity of insulation and h is the external convection heat transfer coefficient.

10-88C It will decrease.

10-89C Yes, the measurements can be right. If the radius of insulation is less than critical radius of insulation of the pipe, the rate of heat loss will increase.

10-90C No.

10-91C For a cylindrical pipe, the critical radius of insulation is defined as $r_{cr} = k/h$. On windy days, the external convection heat transfer coefficient is greater compared to calm days. Therefore critical radius of insulation will be greater on calm days.

10-92 An electric wire is tightly wrapped with a 1-mm thick plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

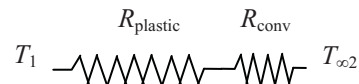
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient accounts for the radiation effects, if any.

Properties The thermal conductivity of plastic cover is given to be $k = 0.15 \text{ W/m}\cdot\text{C}$.

Analysis In steady operation, the rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = VI = (8 \text{ V})(13 \text{ A}) = 104 \text{ W}$$

The total thermal resistance is



$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(24 \text{ W/m}^2 \cdot \text{C})[\pi(0.0042 \text{ m})(10 \text{ m})]} = 0.3158 \text{ }^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2.1/1.1)}{2\pi(0.15 \text{ W/m}\cdot\text{C})(10 \text{ m})} = 0.0686 \text{ }^\circ\text{C/W}$$

$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} = 0.3158 + 0.0686 = 0.3844 \text{ }^\circ\text{C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty} + \dot{Q}R_{\text{total}} = 30^\circ\text{C} + (104 \text{ W})(0.3844 \text{ }^\circ\text{C/W}) = \mathbf{70.0^\circ\text{C}}$$

The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.15 \text{ W/m}\cdot\text{C}}{24 \text{ W/m}^2 \cdot \text{C}} = 0.00625 \text{ m} = 6.25 \text{ mm}$$

Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature.