

11-17 Milk in a thin-walled glass container is to be warmed up by placing it into a large pan filled with hot water. The warming time of the milk is to be determined.

Assumptions **1** The glass container is cylindrical in shape with a radius of $r_0 = 3$ cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.

Properties The thermal conductivity, density, and specific heat of the milk at 20°C are $k = 0.598$ W/m $\cdot^\circ\text{C}$, $\rho = 998$ kg/m 3 , and $c_p = 4.182$ kJ/kg $\cdot^\circ\text{C}$ (Table A-15).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{\mathcal{V}}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.03 \text{ m})^2 (0.07 \text{ m})}{2\pi(0.03 \text{ m})(0.07 \text{ m}) + 2\pi(0.03 \text{ m})^2} = 0.01050 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(120 \text{ W/m}^2 \cdot ^\circ\text{C})(0.0105 \text{ m})}{(0.598 \text{ W/m} \cdot ^\circ\text{C})} = 2.107 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C :

$$b = \frac{hA_s}{\rho c_p \mathcal{V}} = \frac{h}{\rho c_p L_c} = \frac{120 \text{ W/m}^2 \cdot ^\circ\text{C}}{(998 \text{ kg/m}^3)(4182 \text{ J/kg} \cdot ^\circ\text{C})(0.0105 \text{ m})} = 0.002738 \text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38 - 60}{3 - 60} = e^{-(0.002738 \text{ s}^{-1})t} \longrightarrow t = 348 \text{ s} = 5.8 \text{ min}$$

Therefore, it will take about 6 minutes to warm the milk from 3 to 38°C .

