

11-41 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions 1 Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. 2 The thermal properties of the shaft are constant. 3 The heat transfer coefficient is constant and uniform over the entire surface. 4 The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be $k = 14.9 \text{ W/m}\cdot\text{°C}$, $\rho = 7900 \text{ kg/m}^3$, $c_p = 477 \text{ J/kg}\cdot\text{°C}$, $\alpha = 3.95 \times 10^{-6} \text{ m}^2/\text{s}$

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60 \text{ W/m}^2\cdot\text{°C})(0.175 \text{ m})}{(14.9 \text{ W/m}\cdot\text{°C})} = 0.705$$

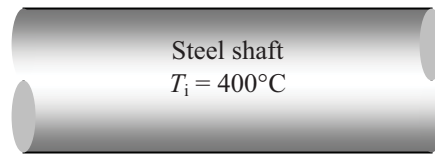
Air
 $T_\infty = 150^\circ\text{C}$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.0904 \quad \text{and} \quad A_1 = 1.1548$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95 \times 10^{-6} \text{ m}^2/\text{s})(20 \times 60 \text{ s})}{(0.175 \text{ m})^2} = 0.1548$$



which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.1548)e^{-(1.0904)^2(0.1548)} = 0.9607$$

$$\frac{T_0 - 150}{400 - 150} = 0.9607 \longrightarrow T_0 = \mathbf{390^\circ\text{C}}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho V = \rho \pi r_o^2 L = (7900 \text{ kg/m}^3)[\pi(0.175 \text{ m})^2(1 \text{ m})] = 760.1 \text{ kg}$$

$$Q_{\max} = mc_p [T_\infty - T_i] = (760.1 \text{ kg})(0.477 \text{ kJ/kg}\cdot\text{°C})(400 - 150)^\circ\text{C} = 90,640 \text{ kJ}$$

Once the constant $J_1 = 0.4679$ is determined from Table 11-3 corresponding to the constant $\lambda_1 = 1.0904$, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2 \left(\frac{T_o - T_\infty}{T_i - T_\infty}\right) \frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2 \left(\frac{390 - 150}{400 - 150}\right) \frac{0.4679}{1.0904} = 0.1761$$

$$Q = 0.1761(90,640 \text{ kJ}) = \mathbf{15,960 \text{ kJ}}$$