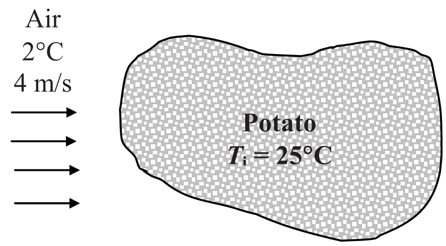
11-57 The center temperature of potatoes is to be lowered to 6°C during cooling. The cooling time and if any part of the potatoes will suffer chilling injury during this cooling process are to be determined.

Assumptions **1** The potatoes are spherical in shape with a radius of r_0 = 3 cm. **2** Heat conduction in the potato is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of potatoes are given to be k = 0.50 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_o}{k} = \frac{(19\text{ W/m}^2.°\text{C})(0.03\text{ m})}{0.5\text{ W/m.°C}} = 1.14$$

From Table 11-2 we read, for a sphere, λ_1 = 1.635 and A_1 = 1.302. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{6-2}{25-2} = 1.302 e^{-(1.635)^2 \tau} \rightarrow \tau = 0.753$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \longrightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.753)(0.03\text{ m})^2}{0.13\times10^{-6}\text{ m}^2/\text{s}} = 5213\text{ s} = \mathbf{1.45\text{ h}}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting, $$\frac{T(r_o)-2}{25-2} = \left(\frac{6-2}{25-2}\right)\frac{\sin(1.635\text{ rad})}{1.635} \longrightarrow T(r_o) = 4.44°\text{C}$$

which is above the temperature range of 3 to 4 °C for chilling injury for potatoes. Therefore, **no part** of the potatoes will experience chilling injury during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{array}{l} \dfrac{1}{\text{Bi}} = \dfrac{k}{hr_o} = \dfrac{0.50\text{W/m.}^\text{o}\text{C}}{(19\text{W/m}^2.^\text{o}\text{C})(0.03\text{m})} = 0.877 \\ \dfrac{T_0 - T_\infty}{T_i - T_\infty} = \dfrac{6-2}{25-2} = 0.174 \end{array}\right\} \tau = \frac{\alpha \text{t}}{r_o^2} = 0.75 \qquad \text{(Fig. 11-17a)}$$

Therefore, $$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.75)(0.03)^2}{0.13\times10^{-6}\text{m}^2/s} = 5192\text{ s} = \mathbf{1.44\text{ h}}$$

The surface temperature is determined from

$$\left.\begin{array}{l} \dfrac{1}{Bi} = \dfrac{k}{hr_o} = 0.877 \\ \dfrac{r}{r_o} = 1 \end{array}\right\} \frac{T(r) - T_\infty}{T_o - T_\infty} = 0.6 \qquad \text{(Fig.11–17b)}$$

which gives $T_{surface} = T_\infty + 0.6(T_o - T_\infty) = 2 + 0.6(6-2) = 4.4°\text{C}$

The slight difference between the two results is due to the reading error of the charts.