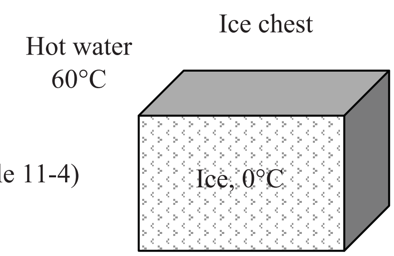
11-73 The outer surfaces of a large cast iron container filled with ice are exposed to hot water. The time before the ice starts melting and the rate of heat transfer to the ice are to be determined.

Assumptions **1** The temperature in the container walls is affected by the thermal conditions at outer surfaces only and the convection heat transfer coefficient outside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the wall are constant.

Properties The thermal properties of the cast iron are given to be $k = 52$ W/m.°C and $\alpha = 1.70\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



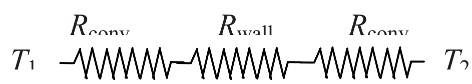
But,

$$\frac{T(x,t)-T_i}{T_s-T_i} = \frac{0.1-0}{60-0} = 0.00167 \rightarrow \quad 0.00167 = erfc(2.226) \quad \text{(Table 11-4)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}} = 2.226 \longrightarrow t = \frac{x^2}{4\times(2.226)^2\alpha} = \frac{(0.05\text{ m})^2}{4(2.226)^2(1.7\times10^{-5}\text{ m}^2/\text{s})} = \mathbf{7.4\ s}$$

The rate of heat transfer to the ice when steady operation conditions are reached can be determined by applying the thermal resistance network concept as



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(250\text{ W/m}^2.°\text{C})(1.2\times2\text{ m}^2)} = 0.00167°\text{C/W}$$

$$R_{wall} = \frac{L}{kA} = \frac{0.05\text{ m}}{(52\text{ W/m.°C})(1.2\times2\text{ m}^2)} = 0.00040°\text{C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(\infty)(1.2\times2\text{ m}^2)} \cong 0°\text{C/W}$$

$$R_{total} = R_{conv,i} + R_{wall} + R_{conv,o} = 0.00167 + 0.00040 + 0 = 0.00207°\text{C/W}$$

$$\dot{Q} = \frac{T_2-T_1}{R_{total}} = \frac{(60-0)°\text{C}}{0.00207°\text{ C/W}} = \mathbf{28{,}990\ W}$$