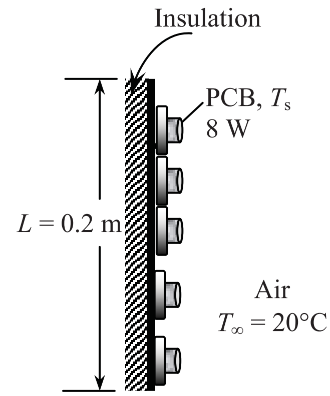
14-29 A printed circuit board (PCB) is placed in a room. The average temperature of the hot surface of the board is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** The heat loss from the back surface of the board is negligible.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (45+20)/2 = 32.5$°C are (Table A-22)

$$k = 0.02607 \text{ W/m.°C}$$
$$\nu = 1.631\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7275$$
$$\beta = \frac{1}{T_f} = \frac{1}{(32.5+273)\text{K}} = 0.003273 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown

(*a*) **Vertical PCB** . We start the solution process by "guessing" the surface temperature to be 45°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the PCB, $L_c = L = 0.2$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003273 \text{ K}^{-1})(45-20 \text{ K})(0.2 \text{ m})^3}{(1.631\times10^{-5} \text{ m}^2/\text{s})^2}(0.7275) = 1.756\times10^7$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.756\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7275}\right)^{9/16}\right]^{8/27}}\right\}^2 = 36.78$$

$$h = \frac{k}{L}Nu = \frac{0.02607 \text{ W/m.°C}}{0.2 \text{ m}}(36.78) = 4.794 \text{ W/m}^2.\text{°C}$$
$$A_s = (0.15 \text{ m})(0.2 \text{ m}) = 0.03 \text{ m}^2$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$8 \text{ W} = (4.794 \text{ W/m}^2.\text{°C})(0.03 \text{ m}^2)(T_s - 20)\text{°C} + (0.8)(0.03 \text{ m}^2)(5.67\times10^{-8})\left[(T_s+273)^4 - (20+273 \text{ K})^4\right]$$

Its solution is

$$T_s = \mathbf{46.6°C}$$

which is sufficiently close to the assumed value of 45°C for the evaluation of the properties and *h*.

(*b*) **Horizontal, hot surface facing up** Again we assume the surface temperature to be 45 °C and use the properties evaluated above. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(0.20 \text{ m})(0.15 \text{ m})}{2(0.2 \text{ m}+0.15 \text{ m})} = 0.0429 \text{ m}.$$

Then

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003273 \text{ K}^{-1})(45-20 \text{ K})(0.0429 \text{ m})^3}{(1.631\times10^{-5} \text{ m}^2/\text{s})^2}(0.7275) = 1.728\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.728\times10^5)^{1/4} = 11.01$$

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$$h = \frac{k}{L_c} Nu = \frac{0.02607 \text{ W/m.°C}}{0.0429 \text{ m}}(11.01) = 6.696 \text{ W/m}^2\text{.°C}$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4)$$

$$8 \text{ W} = (6.696 \text{ W/m}^2\text{.°C})(0.03 \text{ m}^2)(T_s - 20)\text{°C} + (0.8)(0.03 \text{ m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273 \text{ K})^4]$$

Its solution is

T_s = **42.6°C**

which is sufficiently close to the assumed value of 45°C in the evaluation of the properties and h.

(c) **Horizontal, hot surface facing down** This time we expect the surface temperature to be higher, and assume the surface temperature to be 50°C. We will check this assumption after obtaining result and repeat calculations with a better assumption, if necessary. The properties of air at the film temperature of (50+20)/2=35°C are (Table A-22)

$$k = 0.02625 \text{ W/m.°C}$$

$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7268$$

$$\beta = \frac{1}{T_f} = \frac{1}{(35+273)\text{K}} = 0.003247 \text{ K}^{-1}$$

The characteristic length in this case is, from part (b), L_c = 0.0429 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003247 \text{ K}^{-1})(50-20 \text{ K})(0.0429 \text{ m})^3}{(1.655\times10^{-5} \text{ m}^2\text{/s})^2}(0.7268) = 200{,}200$$

$$Nu = 0.27Ra^{1/4} = 0.27(200{,}200)^{1/4} = 5.711$$

$$h = \frac{k}{L_c} Nu = \frac{0.02625 \text{ W/m.°C}}{0.0429 \text{ m}}(5.711) = 3.494 \text{ W/m}^2\text{.°C}$$

Considering both natural convection and radiation heat loses

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4)$$

$$8 \text{ W} = (3.494 \text{ W/m}^2\text{.°C})(0.03 \text{ m}^2)(T_s - 20)\text{°C} + (0.8)(0.03 \text{ m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273 \text{ K})^4]$$

Its solution is

T_s = **50.3°C**

which is very close to the assumed value. Therefore, there is no need to repeat calculations.

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