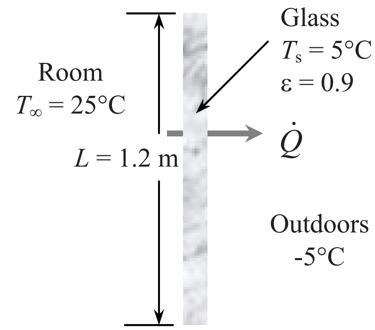
14-37 A glass window is considered. The convection heat transfer coefficient on the inner side of the window, the rate of total heat transfer through the window, and the combined natural convection and radiation heat transfer coefficient on the outer surface of the window are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (5+25)/2 = 15°C$ are (Table A-22)

$$k = 0.02476 \text{ W/m.°C}$$
$$\nu = 1.470\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7323$$
$$\beta = \frac{1}{T_f} = \frac{1}{(15+273)\text{K}} = 0.003472 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the window, $L_c = L = 1.2$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003472 \text{ K}^{-1})(25-5 \text{ K})(1.2 \text{ m})^3}{(1.470\times10^{-5} \text{ m}^2/\text{s})^2}(0.7323) = 3.989\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.989\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7323}\right)^{9/16}\right]^{8/27}}\right\}^2 = 189.7$$

$$h = \frac{k}{L}Nu = \frac{0.02476 \text{ W/m.°C}}{1.2 \text{ m}}(189.7) = \mathbf{3.915\ W/m^2.°C}$$

$$A_s = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

(*b*) The sum of the natural convection and radiation heat transfer from the room to the window is

$$\dot{Q}_{\text{convection}} = hA_s(T_\infty - T_s) = (3.915 \text{ W/m}^2.°\text{C})(2.4 \text{ m}^2)(25-5)°\text{C} = 187.9 \text{ W}$$

$$\dot{Q}_{\text{radiation}} = \varepsilon A_s \sigma(T_{surr}^{\ 4} - T_s^{\ 4})$$
$$= (0.9)(2.4 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(25+273 \text{ K})^4 - (5+273 \text{ K})^4] = 234.3 \text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{radiation}} = 187.9 + 234.3 = \mathbf{422.2\ W}$$

(*c*) The outer surface temperature of the window can be determined from

$$\dot{Q}_{\text{total}} = \frac{kA_s}{t}(T_{s,i} - T_{s,o}) \longrightarrow T_{s,o} = T_{s,i} - \frac{\dot{Q}_{\text{total}}t}{kA_s} = 5°\text{C} - \frac{(422.2 \text{ W})(0.006 \text{ m})}{(0.78 \text{ W/m.°C})(2.4 \text{ m}^2)} = 3.65°\text{C}$$

Then the combined natural convection and radiation heat transfer coefficient on the outer window surface becomes

$$\dot{Q}_{\text{total}} = h_{\text{combined}}A_s(T_{s,o} - T_{\infty,o})$$

or $$h_{\text{combined}} = \frac{\dot{Q}_{\text{total}}}{A_s(T_{s,o} - T_{\infty,o})} = \frac{422.2 \text{ W}}{(2.4 \text{ m}^2)[3.65-(-5)]°\text{C}} = \mathbf{20.35\ W/m^2.°C}$$

Note that $\Delta T = \dot{Q}R$ and thus the thermal resistance R of a layer is proportional to the temperature drop across that layer. Therefore, the fraction of thermal resistance of the glass is equal to the ratio of the temperature drop across the glass to the overall temperature difference,

$$\frac{R_{\text{glass}}}{R_{\text{total}}} = \frac{\Delta T_{\text{glass}}}{\Delta TR_{\text{total}}} = \frac{5-3.65}{25-(-5)} = 0.045 \quad (\text{or } 4.5\%)$$

which is low. Thus it is reasonable to neglect the thermal resistance of the glass.