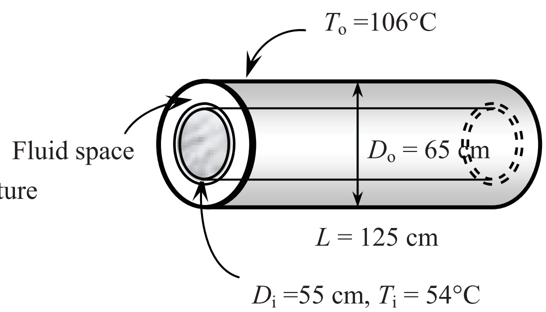
14-76 The space between the two concentric cylinders is filled with water or air. The rate of heat transfer from the outer cylinder to the inner cylinder by natural convection is to be determined for both cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm. **4** Heat transfer by radiation is negligible.

Properties The properties of water air at the average temperature of $(T_i+T_o)/2$ = (54+106)/2 =80°C are (Table A-15)

$$k = 0.670 \text{ W/m.°C}$$
$$\nu = 3.653\times10^{-7} \text{ m}^2/\text{s}$$
$$\text{Pr} = 2.22$$
$$\beta = 0.653\times10^{-3} \text{ K}^{-1}$$



The properties of air at 1 atm and the average temperature of $(T_i+T_o)/2$ = (54+106)/2 = 80°C are (Table A-22)

$$k = 0.02953 \text{ W/m.°C}$$
$$\nu = 2.097\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7154$$
$$\beta = \frac{1}{T_f} = \frac{1}{(80+273)\text{K}} = 0.002833 \text{ K}^{-1}$$

Analysis (*a*) The fluid is water:

$$L_c = \frac{D_o - D_i}{2} = \frac{65-55}{2} = 5 \text{ cm.}$$

$$Ra = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.653\times10^{-3} \text{ K}^{-1})(106-54)\text{K}(0.05 \text{ m})^3}{(3.653\times10^{-7} \text{ m}^2/\text{s})^2}(2.22) = 6.927\times10^8$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.65 \text{ m}}{0.55 \text{ m}}\right]^4}{(0.05 \text{ m})^3\left[(0.55 \text{ m})^{-7/5}+(0.65 \text{ m})^{-7/5}\right]^5} = 0.04136$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.670 \text{ W/m.°C})\left(\frac{2.22}{0.861+2.22}\right)^{1/4}\left[(0.04136)(6.927\times10^8)\right]^{1/4} = 17.43 \text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_0 - T_i) = \frac{2\pi(17.43 \text{ W/m.°C})}{\ln\left(\dfrac{0.65 \text{ m}}{0.55 \text{ m}}\right)}(106-54) = 34{,}090 \text{ W} = \mathbf{34.1\ kW}$$

(*b*) The fluid is air:

$$Ra = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002833 \text{ K}^{-1})(106-54)\text{K}(0.05 \text{ m})^3}{(2.097\times10^{-5} \text{ m}^2/\text{s})^2}(0.7154) = 2.939\times10^5$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln \dfrac{D_o}{D_i}\right]^4}{L_c^3 (D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln \dfrac{0.65\text{ m}}{0.55\text{ m}}\right]^4}{(0.05\text{ m})^3 \left[(0.55\text{ m})^{-7/5} + (0.65\text{ m})^{-7/5}\right]^5} = 0.04136$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02953\text{ W/m.°C})\left(\frac{0.7154}{0.861+0.7154}\right)^{1/4}\left[(0.04136)(2.939\times10^5)\right]^{1/4} = 0.09824\text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_o - T_i) = \frac{2\pi(0.09824\text{ W/m.°C})}{\ln\left(\dfrac{0.65\text{ m}}{0.55\text{ m}}\right)}(106-54) = \mathbf{192\ W}$$