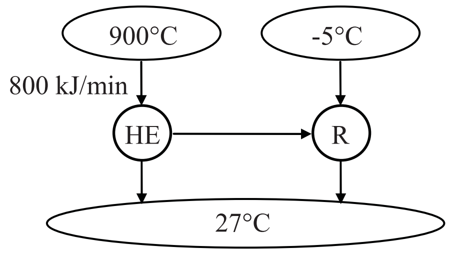
7-111 A Carnot heat engine is used to drive a Carnot refrigerator. The maximum rate of heat removal from the refrigerated space and the total rate of heat rejection to the ambient air are to be determined.

Assumptions The heat engine and the refrigerator operate steadily.

Analysis (a) The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ K}}{1173 \text{ K}} = 0.744$$



Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{\text{net,out}} = \eta_{\text{th}} \dot{Q}_H = (0.744)(800 \text{ kJ/min}) = 595.2 \text{ kJ/min}$$

which is also the power input to the refrigerator, $\dot{W}_{\text{net,in}}$.

The rate of heat removal from the refrigerated space will be a maximum if a Carnot refrigerator is used. The COP of the Carnot refrigerator is

$$\text{COP}_{\text{R,rev}} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(27 + 273 \text{ K})/(-5 + 273 \text{ K}) - 1} = 8.37$$

Then the rate of heat removal from the refrigerated space becomes

$$\dot{Q}_{L,\text{R}} = (\text{COP}_{\text{R,rev}})(\dot{W}_{\text{net,in}}) = (8.37)(595.2 \text{ kJ/min}) = \mathbf{4982 \text{ kJ/min}}$$

(b) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,\text{HE}}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,\text{R}}$),

$$\dot{Q}_{L,\text{HE}} = \dot{Q}_{H,\text{HE}} - \dot{W}_{\text{net,out}} = 800 - 595.2 = 204.8 \text{ kJ/min}$$

$$\dot{Q}_{H,\text{R}} = \dot{Q}_{L,\text{R}} + \dot{W}_{\text{net,in}} = 4982 + 595.2 = 5577.2 \text{ kJ/min}$$

and

$$\dot{Q}_{\text{ambient}} = \dot{Q}_{L,\text{HE}} + \dot{Q}_{H,\text{R}} = 204.8 + 5577.2 = \mathbf{5782 \text{ kJ/min}}$$