ENSC 388 Week #10, Tutorial #8 – Transient Conduction

Problem

A new process for treatment of a special material is to be evaluated. The material, a sphere of radius $r_o = 5mm$, is initially in equilibrium at $400°C$ in a furnace. It is suddenly removed from the furnace and subjected to a two-step cooling process.

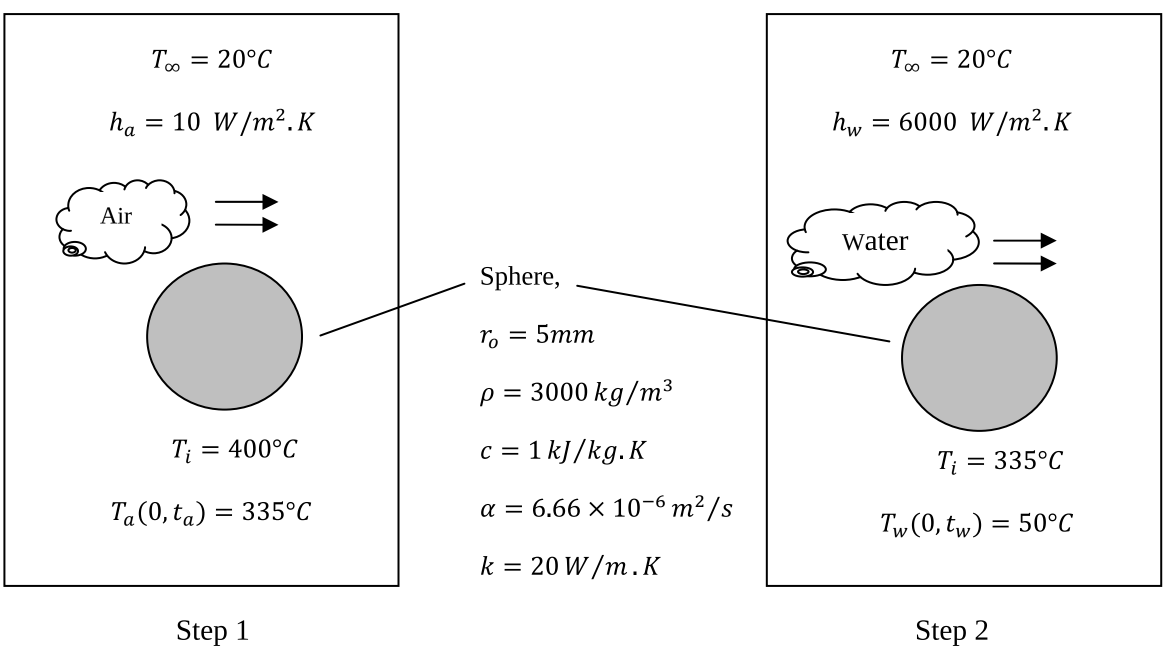
Step 1 Cooling in air at $20°C$ for a period of time t_a until the center temperature reaches a critical value, $T_a(0,t_a)=335°C$. For this situation, the convective heat transfer coefficient is $h_a = 10\ W/m^2.K$.

After the sphere has reached this critical temperature, the second step is initiated.

Step 2 Cooling in a well-stirred water bath at $20°C$, with a convective heat transfer coefficient of $h_w=6000\ W/m^2.K$.

The thermophysical properties of the material are $\rho = 3000\ kg/m^3$, $k=20\ W/m.K$, $c=1000\ J/kg.K$, and $\alpha = 6.66\times10^{-6}\ m^2/s$.

1. Calculate the time t_a required for step 1 of the cooling process to be completed.
2. Calculate the time t_w required during step 2 of the process for the center of the sphere to cool from $335°C$ (the condition at the completion of step 1) to $50°C$.



Solution

Known: Temperature requirements for cooling a sphere.

Find:

1. Time t_a required to accomplish desired cooling in air.
2. Time t_w required to complete cooling in water bath.

Assumptions:

1. One-dimensional conduction in *r*.
2. Constant properties.

Analysis:

1. To determine whether the lumped capacitance method can be used, the Biot number is calculated. From Equation (11-9), with $L_c = r_o/3$,

$$Bi = \frac{h_a r_o}{3k} = \frac{10\,W/m^2.K \times 0.005\,m}{3 \times 20\,W/m.K} = 8.33 \times 10^{-4}$$

Accordingly, the lumped capacitance method may be used, and the temperature is nearly uniform throughout the sphere. From Equation (11-3) it follows that

$$t_a = \frac{\rho V c}{h_a A_s} \ln\frac{\theta_i}{\theta_a} = \frac{\rho r_o c}{3h_a} \ln\frac{T_i - T_\infty}{T_a - T_\infty} \qquad \text{Eq. (1)}$$

where $V=(4/3)\pi r_o^3$ and $A_s=4\pi r_o^2$. Hence

$$t_a = \frac{3000\,kg/m^3 \times 0.005m \times 1000\,J/kg.K}{3 \times 10\,W/m^2.K} ln\frac{400-20}{335-20} = 94\,s$$

2. To determine whether the lumped capacitance method may also be used for the second step of the cooling process, the Biot number is again calculated. In this case

$$Bi = \frac{h_w r_o}{3k} = \frac{6000\ W/m^2.K \times 0.005\,m}{3 \times 20\ W/m.K} = 0.50$$

And the lumped capacitance method is not appropriate. However, to an excellent approximation, the temperature of the sphere is uniform at $t = t_a$ and the one-term approximation may be used for the calculations from $t = t_a$ to $t = t_a + t_w$. The time at which the center temperature reaches $50°C$, that is, $T(0,t_w)=50°C$, can be obtained by rearranging Equation (11-28)

$$Fo = -\frac{1}{\lambda_1^2} ln\left[\frac{\theta_o^*}{A_1}\right] = -\frac{1}{\lambda_1^2} ln\left[\frac{1}{A_1} \times \frac{T(0,t_w) - T_\infty}{T_i - T_\infty}\right] \qquad \text{Eq. (2)}$$

where $t_w = For_o^2/\alpha$. With the Biot number now defined as

$$Bi = \frac{h_w r_o}{k} = \frac{6000\, W/m^2\,.K \times 0.005\, m}{20\, W/m\,.K} = 1.5$$

Table 11-2 yields A_1=*1.376* and λ_1 =*1.800 rad*. It follows that

$$Fo = -\frac{1}{(1.800\, rad)^2} ln\left[\frac{1}{1.376} \times \frac{(50-20)℃}{(335-20)℃}\right] = 0.82$$

and

$$t_w = Fo\frac{r_o^2}{\alpha} = 0.82\frac{(0.005\, m)^2}{6.66 \times \frac{10^{-6} m^2}{s}} = 3.1\, s$$

Note that, with *Fo = 0.82*, use of the one-term approximation is justified.

Comments:

1. If the temperature distribution in the sphere at the conclusion of step 1 were not uniform, the one-term approximation could not be used for the calculations of step 2.
2. The surface temperature of the sphere at the conclusion of step 2 may be obtained from Equation (11-29). With θ_o^*=*0.095* and r^*=*1*,

$$\theta^*(r_o) = \frac{T(r_o) - T_\infty}{T_i - T_\infty} = \frac{0.095}{1.800\, rad}\sin(1.800\, rad) = 0.0514$$

   and

$$T(r_o) = 20℃ + 0.0514(335-20)℃ = 36℃$$

   The infinite series, in Table 11-1 for sphere, and its one-term approximation, Equation (11-29), may be used to compute the temperature at any location in the sphere and at any time $t > t_a$. For $(t - t_a)$ < $0.2(0.005\ m)^2/6.66\times10^{-6}\ m^2/s = 0.75\ s$, a sufficient number of terms must be retained to ensure convergence of the series. For $(t - t_a)$ > *0.75 s*, satisfactory convergence is provided by the one-term approximation.
3. The Heisler charts could also be used to analyze the step 2 process. With Bi^{-1} =*0.67* and θ_o^* =*0.095*, Figure 11-17(a) yields *Fo ≈ 0.8*, in which case t_w ≈*3.0 s*. From Figure 11-17(b), with $r^*=1$, $\theta(r_o)/\theta_o \approx 0.52$, in which case $T(r_o) \approx 20°C + 0.52(50\text{-}20)°C \approx 36°C$.