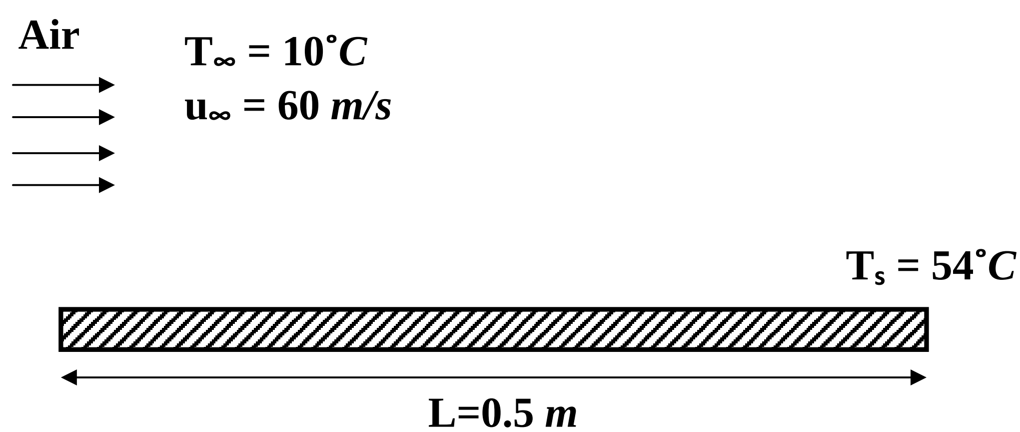
ENSC 388 Week # 10, Tutorial # 9–Heat Transfer Coefficient

Problem 1: Air flows over the top and bottom surfaces of a thin, square plate. The flow regime and the total heat transfer rate are to be determined and the average gradients of the velocity and temperature at the surface are to be estimated.



<u>Solution</u>

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

a) Flow regime.
b) Total Heat Transfer Coefficient.
c) Average gradients of the velocity at the surface.
d) Average gradients of the temperature at the surface.

Step 2: Read properties from the corresponding table

The properties of air at the film temperature (Table A-2)

$$T_f = \frac{T_s + T_\infty}{2} = 32\ [°\mathrm{C}]$$

$\rho = 1.156 \left[\frac{m^3}{kg}\right]$

$c_p = 1.007 \left[\frac{kJ}{kg°C}\right]$

$\upsilon = 1.627 \times 10^5 \left[\frac{m^2}{s}\right]$

$k = 0.02603 \left[\frac{W}{mK}\right]$

$Pr = 0.7276$

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) Steady operating conditions exist.
2) The critical Reynolds number is $Re_{cr} = 5\times10^5$.
3) Radiation effects are negligible.

Step 4: Calculations

(a) The Reynolds number is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(60\ \mathrm{m/s})(0.5\ \mathrm{m})}{1.627\times10^{-5}\ \mathrm{m^2/s}} = 1.844\times10^6$$

which is greater than the critical Reynolds number. Thus we have turbulent flow at the end of the plate.

(*b*) We use modified Reynolds analogy to determine the heat transfer coefficient and the rate of heat transfer

$$\tau_s = \frac{F}{A} = \frac{1.5\ \mathrm{N}}{2(0.5\ \mathrm{m})^2} = 3\left[\frac{\mathrm{N}}{\mathrm{m^2}}\right]$$

$$C_f = \frac{\tau_s}{0.5\rho V^2} = \frac{3\ \mathrm{N/m^2}}{0.5(1.156\ \mathrm{kg/m^3})(60\ \mathrm{m/s})^2} = 1.442\times10^{-3}$$

$$\frac{C_f}{2} = \mathrm{St}\,\mathrm{Pr}^{2/3} = \frac{\mathrm{Nu}_L}{\mathrm{Re}_L\ \mathrm{Pr}}\mathrm{Pr}^{2/3} = \frac{\mathrm{Nu}_L}{\mathrm{Re}_L\ \mathrm{Pr}^{1/3}}$$

$$\text{Nu} = \text{Re}_L \ \text{Pr}^{1/3} \ \frac{C_f}{2} = (1.844\times10^6)(0.7276)^{1/3} \ \frac{(1.442\times10^{-3})}{2} = 1196$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.02603\,\text{W/m.°C}}{0.5\,\text{m}}(1196) = 62.26 \ \left[\frac{\text{W}}{\text{m}^2\text{°C}}\right]$$

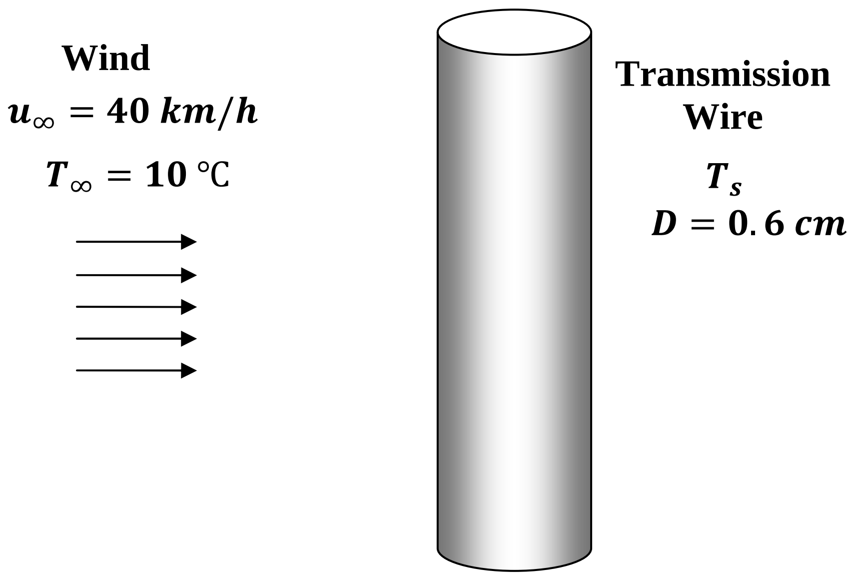
$$\dot{Q} = hA_s(T_s - T_\infty) = (62.26\ \text{W/m}^2\,\text{°C})[2\times(0.5\,\text{m})^2](54-10)\text{°C} = 1370\,[\text{W}]$$

(*c*) Assuming a uniform distribution of heat transfer and drag parameters over the plate, the average gradients of the velocity and temperature at the surface are determined to be

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_0 \longrightarrow \left.\frac{\partial u}{\partial y}\right|_0 = \frac{\tau_s}{\rho\nu} = \frac{3\ \text{N/m}^2}{(1.156\ \text{kg/m}^3)(1.627\times10^{-5}\ \text{m}^2/\text{s})} = 1.60\times10^5\ [\text{s}^{-1}]$$

$$h = \frac{-k\left.\dfrac{\partial T}{\partial y}\right|_0}{T_s - T_\infty} \longrightarrow \left.\frac{\partial T}{\partial y}\right|_0 = \frac{-h(T_s - T_\infty)}{k} = \frac{(62.26\ \text{W/m}^2\cdot\text{°C})(54-10)\text{°C}}{0.02603\ \text{W/m}\cdot\text{°C}} = 1.05\times10^5\left[\frac{\text{°C}}{\text{m}}\right]$$

Problem 2: The wind is blowing across the wire of a transmission line. The surface temperature of the wire is to be determined.



Solution

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

a) Surface temperature of the wire.

Step 2: Read properties from the corresponding table

We assume the film temperature to be 10°C. The properties of air at the film temperature (Table A-2)

$\rho = 1.246 \left[\frac{m^3}{kg}\right]$

$\upsilon = 1.426 \times 10^{-5} \left[\frac{m^2}{s}\right]$

$k = 0.02439 \left[\frac{W}{mK}\right]$

$Pr = 0.7336$

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) Steady operating conditions exist.
2) Radiation effects are negligible.
3) Air is an ideal gas with constant properties.
4) The local atmospheric pressure is 1 atm.

Step 4: Calculations

The Reynolds number is

$$\text{Re} = \frac{u_\infty D}{\nu} = \frac{\left[(40\times 1000/3600)\ \text{m/s}\right](0.006\,\text{m})}{1.426\times 10^{-5}\ \text{m}^2/\text{s}} = 4675$$

The Nusselt number corresponding to this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\ \text{Pr}^{1/3}}{\left[1+\left(0.4/\text{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4675)^{0.5}(0.7336)^{1/3}}{\left[1+\left(0.4/0.7336\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{4675}{282{,}000}\right)^{5/8}\right]^{4/5} = 36.0$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02439\ \text{W/m.°C}}{0.006\,\text{m}}(36.0) = 146.3\left[\frac{\text{W}}{\text{m}^2\,°\text{C}}\right]$$

The rate of heat generated in the electrical transmission lines per meter length is

$$\dot{W} = \dot{Q} = I^2 R = (50\ \text{A})^2(0.002\ \text{Ohm}) = 5.0\,[\text{W}]$$

The entire heat generated in electrical transmission line has to be transferred to the ambient air. The surface temperature of the wire then becomes

$$A_s = \pi DL = \pi(0.006\text{ m})(1\text{ m}) = 0.01885\,[\text{m}^2]$$

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 10°\text{C} + \frac{5\text{ W}}{(146.3\text{ W/m}^2.°\text{C})(0.01885\text{ m}^2)} = 11.8[°\text{C}]$$