ENSC 461

Assignment #2 (Cycles)

Assignment date: Tuesday Jan 23, 2011

Problem 1: (the Stirling cycle)

Show that for an idealized Stirling cycle, the thermal efficiency is:

$$\eta_{th} = 1 - \frac{T_L}{T_H}$$

Problem 2: (Otto cycle)

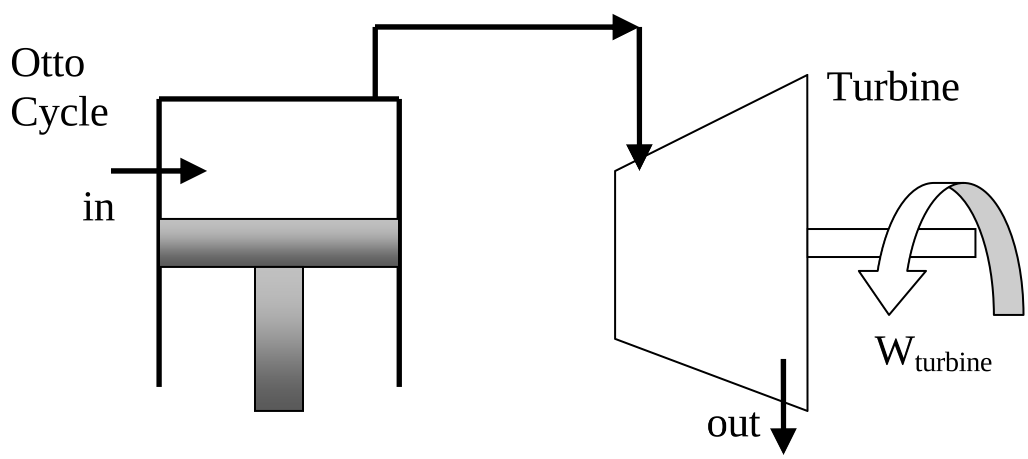
An open, ideal Otto-cycle engine has a compression ratio of 10:1. The air just prior to the compression stroke is at 20C and 100 *kPa*. The maximum cycle temperature is 2000 C. The thermal efficiency of the ideal Otto cycle is 0.60.

Rather than simply discharging the air to the atmosphere after expansion in the cylinder, an isentropic turbine is installed in the exhaust to produce additional work. Assume constant specific heats, the mass flow rate through the turbine is steady and the pressure at the inlet to the cylinder is identical to the pressure at the discharge of the turbine.

i) draw a *T-s* diagram process for the compound engine

ii) determine the work output of the turbine, (*kJ/kg*)

iii) determine the overall thermal efficiency of the compound engine.



Problem 1 Solution:

For idealized Stirling cycle with perfect regeneration, one can write:

$$Q_H = T_H(s_2 - s_1)$$
$$Q_L = T_L(s_3 - s_4)$$

Thermal efficiency is defined:

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L(s_3 - s_4)}{T_H(s_2 - s_1)} \quad (1)$$

From Gibb's equation:

$$Tds = du + Pdv = c_v dT + Pdv$$

If T = const.

$$Tds = Pdv \qquad \text{using ideal gas equation of state}$$
$$ds = \frac{Rdv}{v}$$

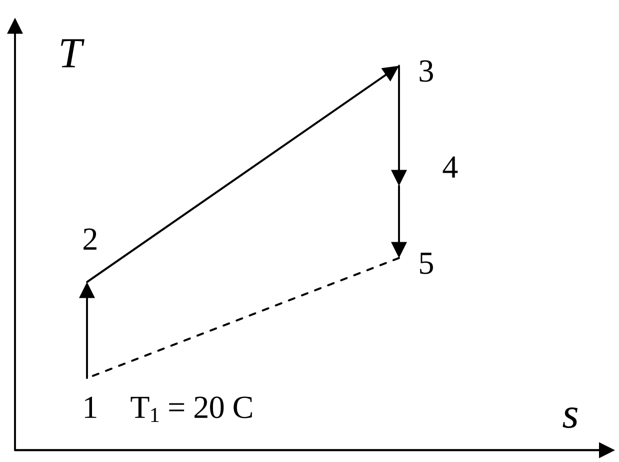
Integrating gives:

$$s_3 - s_4 = R\ln\left(\frac{v_3}{v_4}\right) = R\ln\left(\frac{v_2}{v_1}\right) = s_2 - s_1$$

Therefore, $s_3 - s_4 = s_2 - s_1$; and Eq. (1) gives:

$$\eta_{th} = 1 - \frac{T_L}{T_H}$$

Problem 2 Solution:



Part i)

For an isentropic process

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{(k-1)} \quad \text{and} \quad \frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{(k-1)} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

Therefore;

T_2 = 293 K (10) $^{1.4-1}$ = 735.98 K

and

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{(k-1)} = T_3\left(\frac{v_4}{v_3}\right)^{-(k-1)}$$

For an IC engine,

$$\frac{v_4}{v_3} = \frac{v_1}{v_2}$$

Therefore

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{(k-1)} = T_3\left(\frac{v_1}{v_2}\right)^{-(k-1)} = (2000+273)K \times (10)^{-(1.4-1)} = 904.9K$$

The net work output from the Otto cycle is

$w_{net,\,Otto} = c_p\,(T_3 - T_4) - c_p\,(T_2 - T_1)$

= 1.005 kJ/kg.K (2273 - 904.9 – 735.98 + 293) K

= 929.75 kJ/kg

We have an isentropic process between 3 and 5, we can write:

$$\frac{T_5}{T_3} = \left(\frac{P_5}{P_3}\right)^{(k-1)/k}$$

Since the air behaves as an ideal gas, and we know that $P_1 = P_5 = P_{atm}$, we can write

$P_3 = R\,T_3 / v_3$

$P_1 = P_5 = R\,T_1 / v_1$

Therefore

$$\frac{T_5}{T_3} = \left(\frac{P_5}{P_3}\right)^{(k-1)/k} = \left(\frac{R.T_1}{v_1}\frac{v_3}{R.T_3}\right)^{(k-1)/k}$$

But for an Otto cycle, we have constant volume heat addition between 2 and 3 and $v_2 = v_3$

$$\frac{T_5}{T_3} = \left(\frac{T_1}{v_1}\frac{v_2}{T_3}\right)^{(k-1)/k} = \left(\frac{T_1}{T_3}\frac{v_2}{v_1}\right)^{(k-1)/k} = (2273K)\left(\frac{473K}{2273K}\frac{1}{10}\right)^{0.4/1.4} = 655.7K$$

Part ii)

The work output of the turbine is

$w_{turbine} = (h_4 – h_5) = c_p (T_4 – T_5)$

$= 1.005$ kJ/kg.K (904.9 – 655.7) K

$= 250.4$ kJ/kg

Part iii)

The thermal efficiency of the compound engine is given by

$$\eta_{th} = \frac{w_{net,Otto} + w_{turbine}}{q_h}$$

$$q_h = w_{net,Otto} / \eta_{Otto} = 929.75(kJ/kg)/0.6 = 1549.58(kJ/kg)$$

thus

$$\eta_{th} = (929.75 kJ/kg + 250.4 kJ/kg)/1549.58 kJ/kg = 0.762$$