ENSC 461

Assignment #4 (Gas-turbine)

Assignment date: Tuesday Feb. 08, 2011

Due date: Tuesday Feb. 15, 2011

Problem 1:

An ideal gas-turbine cycle with ideal regeneration and with *n*-stage of compression (with intercooling) and expansion (with reheating), equal increment at each stage of compression and expansion, has an overall pressure ratio of *r*. Air enters each stage of the compressor at T_1 and each stage of the turbine at T_3.

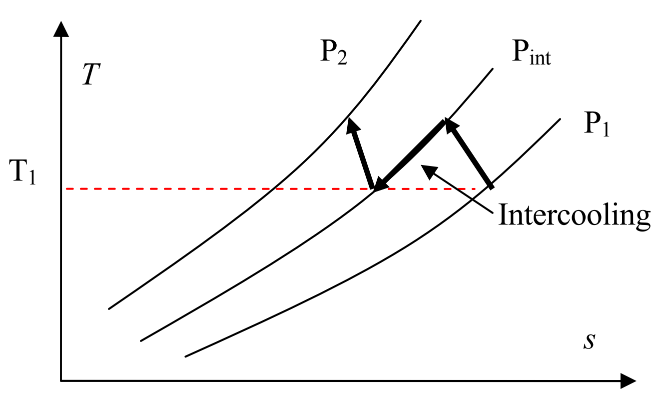
a) Derive relationships for exit temperature at each stage of compression and expansion. Compare the results when $n = 1$ with the single-stage ideal Brayton cycle which operates at the same pressure ratio and works between the same min and max temperatures T_1 and T_3.

b) Determine the thermal efficiency for the *n*-stage system. Plot a *T-s* diagram for the system. Compare the thermal efficiency of the n-stage cycle, when $n \rightarrow \infty$ with the thermal efficiency of a Carnot engine working between the same temperatures T_1 and T_3. Elaborate on your answer.

Problem 2:

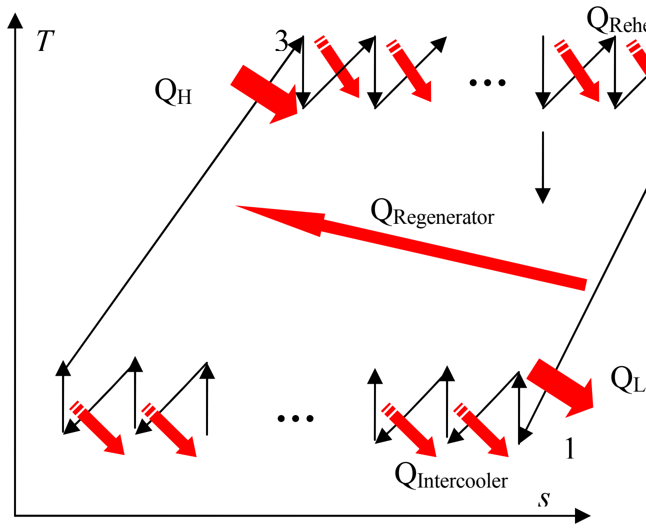
Show that the work input to a two-stage compressor is minimized when equal pressure ratios are maintained across each stage. That is:

$$P_{int} = \sqrt{P_1 P_2}$$



Note: assume polytropic (more general than isentropic) compression.

Solution, Problem 1:



Compression:

Since pressure increments are identical, one can write:

$P_{i,c} = P_1 + (i-1)\,(P_2 - P_1)\,/\,n \qquad 1 < i < n+1 \quad (1)$

where P_1 and P_2 are the first and last stage pressure of compression, and $P_{i,c}$ is the pressure at i-th stage of compression.

Equation (1) can be re-arranged as

$P_{i,c} = P_1\,[1 + (i-1)\,(r-1)]$

where $r = P_2\,/\,P_1$.

Since the compression process is isentropic, one can write:

$$\frac{T_{i,c}}{T_{i-1,c}} = \left(\frac{P_{i,c}}{P_{i-1,c}}\right)^{(k-1)/k}$$

The inlet temperature to each compression stage is $T_1 = T_{i-1}$;

$$T_{i,c} = T_1\left[\frac{1+\dfrac{i-1}{n}(r-1)}{1+\dfrac{i-2}{n}(r-1)}\right]^{(k-1)/k} \qquad (2)$$

Substituting $n = 1$ ($i = 2$) yields;

$T_2 = T_1\, r^{(k-1)/k}$

Expansion:

Following the same method, for multi-turbine with reheat, we can write:

$P_{i,t} = P_2 + (i-1)\,(P_2 - P_1)\,/\,n \qquad 1 < i < n+1 \quad (3)$

or,

$P_{i,t} = P_2\,[1 + (i\text{-}1)\,(1\text{-}1/r)]$

Since isentropic expansion occurs in turbines, we have

$$\frac{T_{i,t}}{T_{i-1,t}} = \left(\frac{P_{i,c}}{P_{i-1,c}}\right)^{(k-1)/k}$$

which can be written as:

$$T_{i,t} = T_3\left[\frac{1+\dfrac{i-1}{n}\left(1-\dfrac{1}{r}\right)}{1+\dfrac{i-2}{n}\left(1-\dfrac{1}{r}\right)}\right]^{(k-1)/k} \qquad (4)$$

Because of reheating $T_{i-1,t} = T_3$ for all stages of turbine. For example for a single-stage system, Eq. (4) yields:

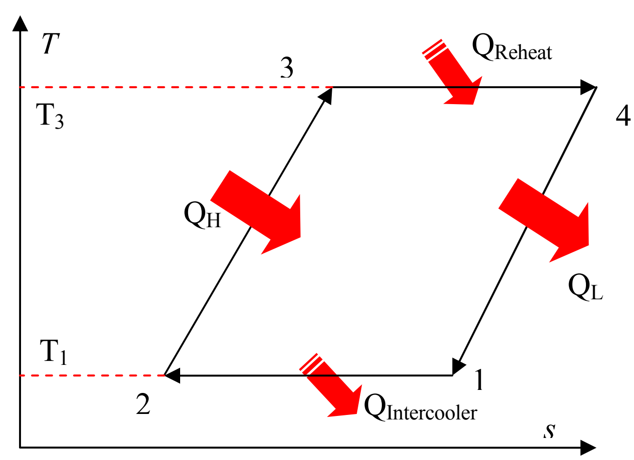
$T_4 = T_3\,(1/r)^{(k-1)/k}$

It should be noted that at the limit when n → ∞, Eqs. (2) and (4) become:

$$\text{limit}_{n\to\infty} T_{i,c} = \text{limit}_{n\to\infty} T_1\left[\frac{1+\dfrac{i-1}{n}(r-1)}{1+\dfrac{i-2}{n}(r-1)}\right]^{(k-1)/k} \rightarrow T_1 \qquad (5)$$

$$\text{limit}_{n\to\infty} \quad T_{i,t} = \text{limit}_{n\to\infty} T_3\left[\frac{1+\dfrac{i-1}{n}\left(1-\dfrac{1}{r}\right)}{1+\dfrac{i-2}{n}\left(1-\dfrac{1}{r}\right)}\right]^{(k-1)/k} \rightarrow T_3 \qquad (6)$$

Equations (5) and (6) show that if the number of compression (with intercooler) and expansion (with reheat) is increased, the ideal cycle of expansion and compression processes become isothermal, see Fig. below.



Therefore, input heat becomes:

$q_{in} = q_{\,reheat}$

Note that at the limit $n \rightarrow \infty$, $q_H = q_L \rightarrow 0$. Also $T_3 = T_4$ and $T_1 = T_2$.

Assuming ideal gas and standard-cold-assumption:

$q_{in} = T_3\,(s_4 - s_3)$

$s_4 - s_3 = - R \ln (P_4 / P_3) = - R \ln (1/r) = R \ln (r)$

$q_{in} = R\, T_3 \ln (r)$ (7)

The heat-rejection from the system is:

$q_{out} = q_{\,intercooling}$

Assuming ideal gas and standard-cold-assumption:

$q_{out} = T_1\,(s_1 - s_2)$

$s_1 - s_2 = - R \ln (P_1 / P_2) = - R \ln (1/r) = R \ln (r)$

$q_{in} = R\, T_1 \ln (r)$ (8)

The thermal efficiency for the cycle becomes:

$$\eta_{n-stage,Brayton} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{RT_1 \ln r}{RT_3 \ln r} = 1 - \frac{T_1}{T_3}$$

which is the Carnot cycle thermal efficiency. In fact the T-s diagram ideal gas-turbine with regenerator and n-stage compression and expansion with intercooler and reheat becomes the T-s diagram of Ericson cycle working between the same …

Solution, Problem 2:

The total work input for a two-stage compressor is the sum of the work inputs for each stage of compression:

$$w_{comp,in} = w_{comp1,in} + w_{comp2,in}$$
$$= \frac{nRT_1}{n-1}\left[\left(\frac{P_{\text{int}}}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_{\text{int}}}\right)^{(n-1)/n} - 1\right]$$

The only variable in this equation is P_{int}. The P_{int} that minimizes the total work is determined by differentiating this expression with respect to P_{int} and setting the resulting expression equal to zero. It yields:

$$P_{\text{int}} = \sqrt{P_1 P_2}$$