ENSC 461

Assignment #5 (Exergy)

Assignment date:

Due date:

Problem 1:

A system undergoes a refrigeration cycle while receiving Q_c by heat transfer at temperature T_c and discharging energy Q_H by heat transfer at a higher temperature T_H. There are no other heat transfers.

a) Using an exergy balance, show that the network input to the cycle cannot be zero.

b) Show that the coefficient of performance of the cycle can be expressed as:

$$COP = \left(\frac{T_c}{T_H - T_c}\right)\left(1 - \frac{T_H X_{destroyed}}{T_0 \left(Q_H - Q_c\right)}\right)$$

where $X_{destroyed}$ is the exergy destruction and T_0 is the temperature of the surroundings.

c) Using the result of part (b), obtain an expression for the coefficient of performance.

Problem 2:

Helium gas enters an insulated nozzle operating at steady state at 1300 *K*, 4 bar, and 10 *m/s*. At the exit, the temperature and pressure of the helium are 900 *K* and 1.45 bar, respectively. Determine:

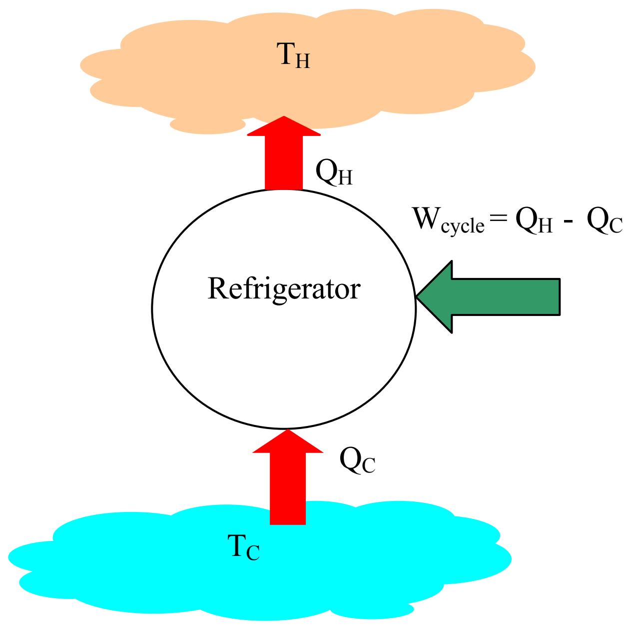
a) the exit velocity in m/s

b) the isentropic nozzle efficiency

c) the rate of exergy destruction, in *kJ/kg* of gas flowing through the nozzle.

Assume the ideal gas model for helium and ignore the effects of gravity. Let T_0 = 20°C and P_0 = 1atm.

Solution, Problem 1:



Assumptions:

1) The system shown undergoes a refrigeration cycle

2) Q_C and Q_H are the only heat transfers and are in the directions of the arrows

3) T_c and T_H are constant and $T_H > T_C$ and the surroundings' temperature is T_0.

Analysis:

a)

An exergy balance for the cycle reads:

$$\Delta X_{cycle} = \left(1 - \frac{T_0}{T_c}\right)Q_c - \left(1 - \frac{T_0}{T_H}\right)Q_H - \left(W_{cycle} - P_0 \underbrace{\Delta V}_{=0}\right) - X_{destroyed} = 0$$

where ΔX_{cycle} and ΔV are zero for a cycle. Introducing the energy balance,

$$Q_H = W_{cycle} + Q_c$$

Since the work is being done on the cycle, it should be considered negative. We get:

$$0 = \left(1 - \frac{T_0}{T_c}\right)Q_c - \left(1 - \frac{T_0}{T_H}\right)\left(W_{cycle} + Q_c\right) - \left(-W_{cycle}\right) - X_{destroyed}$$

$$= T_0\left(\frac{1}{T_H} - \frac{1}{T_c}\right)Q_c + \frac{T_0}{T_H}W_{cycle} - X_{destroyed} \qquad (1)$$

Solving for $X_{destroyed}$, and setting W_{cycle} to zero.

$$X_{destroyed} = \underbrace{T_0}_{>0}\underbrace{\left(\frac{1}{T_H}-\frac{1}{T_c}\right)}_{<0}\underbrace{Q_c}_{>0} + \frac{T_0}{T_H}\underbrace{W_{cycle}}_{=0} \Rightarrow X_{destroyed} < 0 \quad \text{(impossible!)}$$

Thus, W_{cycle} cannot be zero!

b) Solving Eq. (1) for COP, one finds:

$$COP = \frac{Q_c}{W_{cycle}}$$

$$\left(\frac{T_H - T_c}{T_H T_c}\right)Q_c = \frac{W_{cycle}}{T_H} - \frac{X_{destroyed}}{T_0}$$

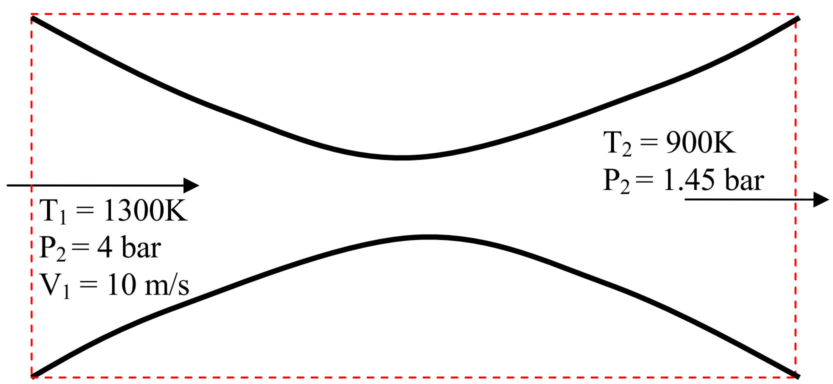
$$COP = \frac{Q_c}{W_{cycle}} = \left(\frac{T_c}{T_H - T_c}\right)\left(1 - \frac{T_H X_{destroyed}}{T_0 W_{cycle}}\right) = \left(\frac{T_c}{T_H - T_c}\right)\left(1 - \frac{T_H X_{destroyed}}{T_0 (Q_H - Q_c)}\right)$$

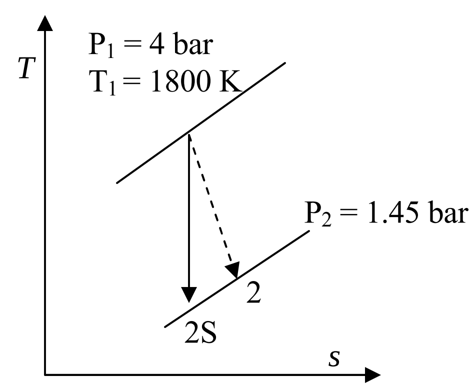
c) From the result of part b), COP increases as $X_{destroyed} \rightarrow 0$. Thus, when $X_{destroyed} = 0$:

$$COP_{\max} = \left(\frac{T_c}{T_H - T_c}\right)$$

As expected.

Solution, Problem 2:





Assumptions:

1) The control volume shown is at steady-state
2) No work or heat transfer occurs in the c.v. shown
3) Potential energy effects are negligible.
4) Helium is modeled as ideal gas.
5) the environment is at T_0 = 20C and P_0 = 1 atm.

Analysis:

Energy balance for the nozzle at steady-state reduces to:

$$0 = \underbrace{\dot{Q}_{cv} + \dot{W}_{cv}}_{=0} + \dot{m}\left(h_1 - h_2 + \left(\frac{V_1^2 - V_2^2}{2} \right) + \underbrace{g\Delta z}_{=0} \right) \quad (1)$$

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

For monatomic gases, such as He, Ne, and Ar, $\bar{c}_p$ over a wide range of temperature range is very nearly equal to 5/2 $\bar{R}$. Thus, $h_1 - h_2 = c_p (T_1 - T_2)$ and:

$$V_2 = \sqrt{\left(10\frac{m}{s}\right)^2 + 2\left[2.5\frac{8314}{4.003}\frac{N.m}{kg.K}\right](1300-900)\left(\frac{1kg.m/s^2}{1N}\right)} = 2038\frac{m}{s}$$

To determine the isentropic nozzle efficiency requires the temperature at state 2s. With k = 1.667, we have:

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1300\left(\frac{1.45}{4}\right)^{\frac{0.667}{1.667}} = 866.2K$$

Then with Eq. (1)

$$V_{2s} = \sqrt{V_1^2 + 2(h_1 - h_{2s})} = \sqrt{V_1^2 + 2c_p(T_1 - T_{2s})} = 2122\frac{m}{s}$$

The isentropic nozzle efficiency is:

$$\eta_{nozzle} = \frac{V_2^2/2}{V_{2s}^2/2} = \left(\frac{2038}{2122}\right)^2 = 0.922 \quad (92.2\%)$$

The exergy destruction given by:

$$\frac{\dot{X}_{destruction}}{\dot{m}} = T_0\left(\frac{\dot{S}_{gen}}{\dot{m}}\right)$$

No heat transfer and work transfer occurs in the nozzle, thus an entropy balance gives:

$$\frac{\dot{X}_{destruction}}{\dot{m}} = T_0(s_2 - s_1) = T_0\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right] = T_0R\left(2.5\ln\frac{T_2}{T_1} - \ln\frac{P_2}{P_1}\right)$$

$$= (293)\left(\frac{8.314}{4.003}\frac{kJ}{kg.K}\right)\left(2.5\ln\frac{900}{1300} - \ln\frac{1.45}{4}\right) = 58.1\left(\frac{kJ}{kg}\right)$$