

ENSC 461

Assignment #7 (Non-Reacting Mixtures and HVAC)

Assignment date:

Due date:

Problem 1:

A system consists, initially of n_A moles of gas A at pressure P and temperature T and n_B moles of gas B separate from gas A but at the same pressure and temperature. The gases are allowed to mix with no heat or work interactions with the surroundings. The final equilibrium pressure and temperature are P and T , respectively, and the mixing occurs with no change in total volume.

- Assuming ideal gas behavior, obtain an expression for the entropy produced in terms of R , n_A , and n_B .
- Using the result of part a), demonstrate that the entropy produced has a positive value.
- Would entropy be produced when samples of the same gas at the same temperature and pressure mix? Explain.

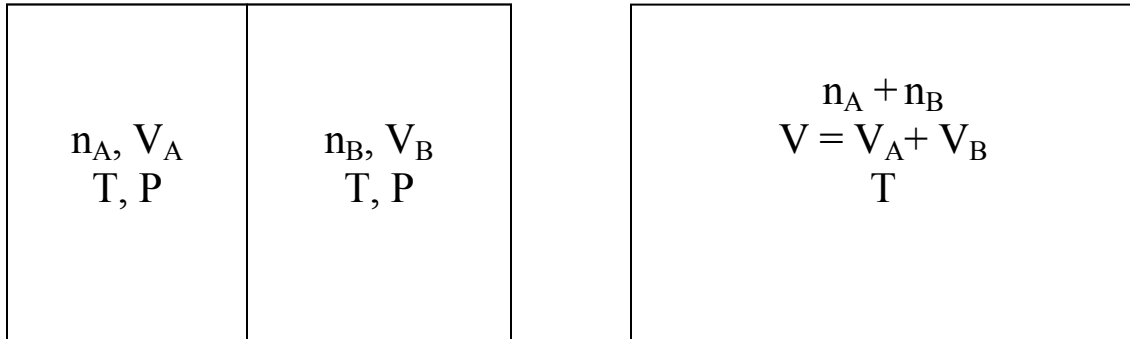
Problem 2:

Air at 35°C, 1 atm, and 10% relative humidity enters an evaporative cooler operating at steady-state. The volumetric flow rate of the incoming air is 50 m³/min. Liquid water at 20°C enters the cooler and fully evaporates. Moist air exits the cooler at 25°C, and 1 atm. If there is no significant heat transfer between the device and its surroundings, show the process on a psychrometric chart and determine:

- The rate at which liquid enters the cooler, in kg/min.
- The relative humidity at the exit.
- The rate of exergy destruction, in kJ/min, for $T_0 = 20^\circ\text{C}$.

Neglect potential and kinetic energy effects. Calculate properties of the moist air using: a) the relationships derived in the class, b) the psychrometric chart. Compare the two sets.

Solution Problem 1



Assumptions:

1. The system consists of gas A and B together.
2. Each gas behaves as an ideal gas. The final mixture also acts as an ideal gas with each component occupying the total volume and exhibiting the mixture temperature.
3. No heat or work interactions with the surroundings occur.

Analysis:

An entropy balance for system reduces to:

$$S_2 - S_1 = \underbrace{\int_1^2 \left(\frac{\delta Q}{T} \right)}_{=0} + S_{gen}$$

where the entropy transfer term drops out in the adiabatic process. The initial entropy of the system is the sum of the entropies of the gases when separate:

$$S_1 = n_A \bar{s}_A(T, P) + n_B \bar{s}_B(T, P)$$

The entropy of the system at the final state equals the sum of the entropies of the gases A and B evaluated at the conditions at which they exist in the mixture:

$$S_2 = n_A \bar{s}_A(T, y_A P) + n_B \bar{s}_B(T, y_B P)$$

Combining the last three equations and solving for the entropy production

$$S_{gen} = n_A [\bar{s}_A(T, y_A P) - \bar{s}_A(T, P)] + n_B [\bar{s}_B(T, y_B P) - \bar{s}_B(T, P)]$$

The specific entropy changes can be evaluated as (no heat transfer):

$$\bar{s}_A(T, y_A P) - \bar{s}_A(T, P) = -\bar{R} \ln \frac{y_A P}{P} = -\bar{R} \ln y_A$$

$$\bar{s}_B(T, y_B P) - \bar{s}_B(T, P) = -\bar{R} \ln \frac{y_B P}{P} = -\bar{R} \ln y_B$$

where $\bar{R} = 8.314 \text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K}$ is the universal constant of gases. Using the above equation in the entropy generation expression gives:

$$S_{gen} = -\bar{R}[n_A \ln y_A + n_B \ln y_B]$$

Since y_A and y_B are each smaller than unity, S_{gen} will be positive in accordance with the second law.

If the gases were identical, the initial entropy of the system would be:

$$S_1 = n_A \bar{s}(T, P) + n_B \bar{s}(T, P)$$

The final entropy of the system would be:

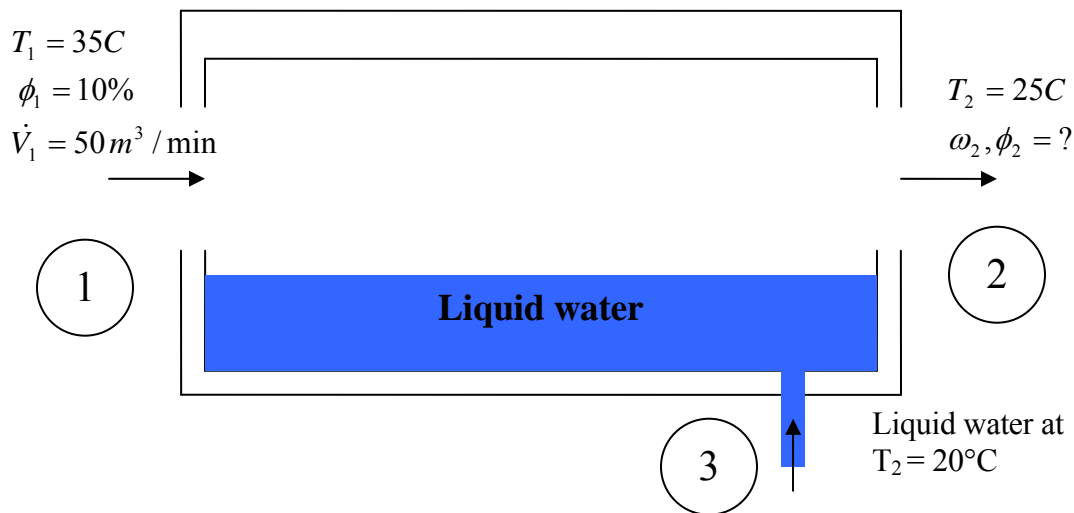
$$S_2 = (n_A + n_B) \bar{s}(T, P)$$

Clearly $S_{gen} = 0$ for this case, i.e., no entropy will be produced.

Solution Problem 2

Assumptions:

- 1- Steady-state operation
- 2- No heat or work interactions with the surroundings.
- 3- All the entering liquid evaporates into the moist air stream.
- 4- The liquid water enters at saturated liquid.
- 5- $T_0 = 293$ K.



Analysis:

At steady-state, mass balance gives:

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a \quad \text{and} \quad \dot{m}_{w1} + \dot{m}_w = \dot{m}_{w2} \quad \text{or}$$

$$\dot{m}_w = \dot{m}_a (\omega_2 - \omega_1)$$

$$\dot{m}_w / \dot{m}_a = (\omega_2 - \omega_1)$$

The relative humidity at point 1 can be found from:

$$P_{v1} = \phi_1 P_g(T_1) = 0.1(0.05628) = 0.005628 \text{ bar}$$

Thus;

$$\omega_1 = 0.622 \left[\frac{0.005628}{1 - 0.005628} \right] = 0.00352 \text{ kg water / kg dry air}$$

ω_2 can be found using an energy balance:

$$0 = \underbrace{\dot{Q}_{CV}}_{=0} - \underbrace{\dot{W}_{CV}}_{=0} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_3 - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

Or

$$0 = \dot{m}_a [(h_{a1} - h_{a2}) + \omega_1 h_{g1} + (\omega_2 - \omega_1) h_{f3} - \omega_2 h_{g2}]$$

$$\omega_2 = \frac{(h_{a1} - h_{a2}) + \omega_1 (h_{g1} - h_{f3})}{h_{g2} - h_{f3}} = \frac{1.005(35 - 25) + 0.00352(2565.3 - 83.96)}{2547.2 - 83.96} = 0.00763 \text{ kg water / kg dry air}$$

The mass flow rate of dry air is:

$$\dot{m}_a = \frac{\dot{V}_1 P_{a1}}{RT_1} = \frac{(50 \text{ m}^3 / \text{min}) (101.325 \text{ kPa})}{\left(\frac{8.314}{28.97}\right) 308 \text{ K}} = 56.32 \text{ kg / min}$$

Thus;

$$\dot{m}_w = \dot{m}_a (\omega_2 - \omega_1) = 0.24 \text{ kg / min}$$

Relative humidity at point 3 can be found from:

$$P_{v2} = \frac{\omega_2 P}{0.622 + \omega_2} = 0.0121 \text{ bar} \Rightarrow \phi_2 = \frac{P_v}{P_g(T_2)} = .382 = 38\%$$

The rate of exergy destruction can be found from:

$$\dot{X}_{destroyed} = T_0 \dot{S}_{gen}$$

An entropy balance gives:

$$\frac{\dot{S}_{gen}}{\dot{m}_a} = [s_a(T_2, P_{a2}) - s_a(T_1, P_{a1})] + \omega_2 s_v(T_2, P_{v2}) - \omega_1 s_v(T_1, P_{v1}) - (\omega_2 - \omega_1) s_{f3}$$

The specific entropy for water-vapor can be found: $s_v(T, P_v) = s_g(T) - R \ln \phi$; the above equation yields:

$$\frac{\dot{S}_{gen}}{\dot{m}_a} = \left[c_{pa} \ln \frac{T_2}{T_1} - R \ln \frac{P_{a2}}{P_{a1}} \right] + \omega_2 [s_g(T_2) - R \ln \phi_2] - \omega_1 [s_g(T_1) - R \ln \phi_1] - (\omega_2 - \omega_1) s_{f3}$$

$$= 0.003 \text{ kJ / kg dry air.K}$$

$$\dot{X}_{destroyed} = 56.26 \text{ kg / min} (293 \text{ K}) (0.003 \text{ kJ / kg dry air.K}) = 49.44 \text{ kJ / min}$$