Exergy

Exergy is the theoretical limit for the work potential that can be obtained from a source or a system at a given state when interacting with a reference (environment) at a constant condition.

A system is said to be in the dead state when it is in thermodynamic equilibrium with its environment. Unless otherwise stated, assume the dead state to be:

$$P_0 = 1\ atm \quad \text{and } T_0 = 25°C$$

A system delivers the maximum possible work as it undergoes a reversible process from the specified initial state to the state of its environment (dead state). This represents the *useful work potential*, or *exergy*, or *availability*.

Exergy is a property of the system-environment combination (not the system alone).

Exergy of Kinetic Energy

Kinetic energy can be converted to work entirely; thus:

$$x_{KE} = ke = \frac{V^2}{2} \quad (kJ/kg)$$

where V is the velocity of the system relative to the environment.

Exergy of Potential Energy

Potential energy is also a form of mechanical energy and can be converted to work entirely; thus:

$$x_{PE} = pe = gz \quad (kJ/kg)$$

Some Definitions

Surroundings work: is the work done by or against the surroundings during a process. This work cannot be recovered and utilized. For a cylinder-piston assembly, one can write:

$$W_{surr} = P_0(V_2 - V_1)$$

The difference between the actual work W and the surroundings work W_{surr} is called the useful W_u:

$$W_u = W - W_{surr} = W - P_0 (V_2 - V_1)$$

Reversible Work: W_{rev} is the max amount of useful work that can be produced (or the min work needs to be supplied) as the system undergoes a process between the initial and final states. When the final state is the dead state, the reversible work equals exergy.

Irreversibility: I is equal to the exergy destroyed; thus for a reversible process the irreversibility or exergy destruction is zero.

$$I = W_{rev,out} - W_{u,out}$$

or,

$$I = W_{u,in} - W_{rev,in}$$

Example 1

A 500-kg iron block is initially at 200°C and is allowed to cool to 27°C by transferring heat to the surroundings air at 27°C.

Determine the reversible work and the irreversibility for this process.

Assumption:

1) the kinetic potential energies are negligible.

2) the process involves no work interactions.

Analysis:

The reversible work is determined by considering a series of imaginary reversible heat engines operating between the source (at a variable temperature T) and the sink T_0.

$$\delta W_{rev} = \eta_{th,rev}\delta Q_{in} = \left(1 - \frac{T_0}{T}\right)\delta Q_{in}$$

$$W_{rev} = \int_{T}^{T_0} \left(1 - \frac{T_0}{T}\right)\delta Q_{in}$$

Using the first law for the iron block, a relationship for the heat transfer can be found:

$$-\delta Q_{out} = dU = mc_{ave}dT$$

then

$$\delta Q_{in} = -mc_{ave}dT$$

The reversible net work is determined to be:

$$W_{rev} = \int_{T_1}^{T_0} \left(1 - \frac{T_0}{T}\right)\left(-mc_{ave}dT\right) = mc_{ave}\left(T_1 - T_0\right) - mc_{ave}T_0 \ln\frac{T_1}{T_0} = 8191 \quad kJ$$

Note that the first term in the above equation, $mc_{ave}\,(T_1 - T_0)$ = 38,925 kJ, is the total heat transfer from the iron block to the engine. This means that only 21% of the heat transferred from the iron block could have been converted to work.

The irreversibility for the process is determined from:

$$I = W_{rev} - W_u = 8191 - 0 = 8191\ kJ$$

The entire work potential is wasted.

The Second Law Efficiency

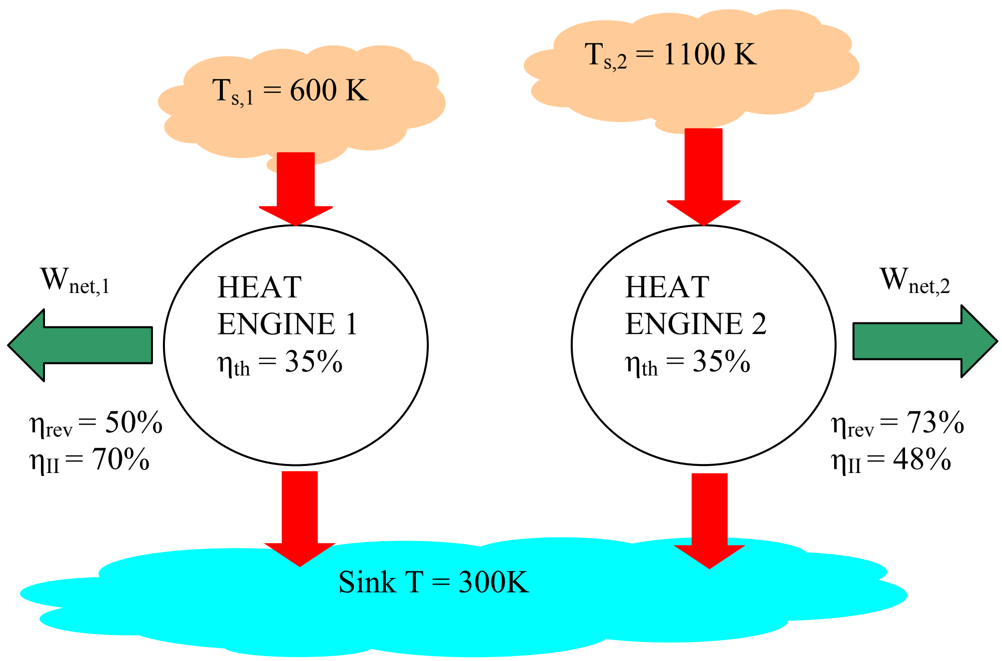
The second law efficiency, η_{II} is the ratio of actual thermal efficiency to the maximum possible (reversible) thermal efficiency under the same conditions:

$$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = 1 - \frac{\text{Exergy destroyed}}{\text{Exergy supplied}}$$

$$\eta_{II} = \frac{\eta_{th}}{\eta_{th,rev}}\left(\text{heat engines}\right) \qquad \eta_{II} = \frac{W_u}{W_{rev}}\left(\text{Work - producing devices}\right)$$

$$\eta_{II} = \frac{W_{rev}}{W_u}\left(\text{Work - consuming devices}\right) \qquad \eta_{II} = \frac{COP}{COP_{rev}}\left(\text{refrigerators and heat pumps}\right)$$

The second law efficiency serves as a measure of approximation to reversible operation; thus its value should range from 0 to 1.



Exergy of A Fixed Mass

Consider a stationary cylinder-piston assembly that contains a fluid of mass *m* at *T, P, U,* and *S*. The system is allowed to undergo a differential change.

The energy balance for the system during this differential process can be expressed as:

$$-\delta Q - \delta W = dU \qquad (1)$$

The system involves some boundary work:

$$\delta W = PdV = \left(P - P_0\right)dV + P_0 dV = \delta W_{b,useful} + P_0 dV \quad (2)$$

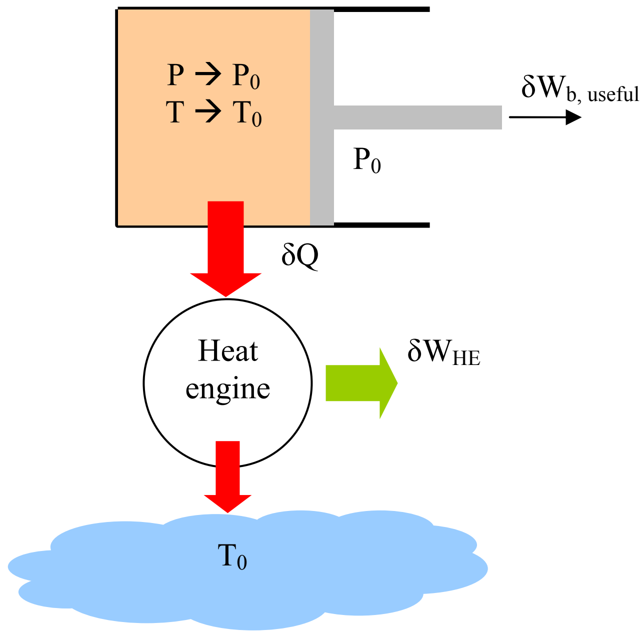
For a system, which is at temperature T, to have a reversible heat transfer with the surroundings at T_0, heat transfer must occur through a reversible heat engine ($\eta_{th} = 1 - T_L/T_H$). For the reversible heat engine, one can write:

$$\delta W_{HE} = \left(1 - \frac{T_0}{T}\right)\delta Q = \delta Q - \delta Q \frac{T_0}{T}$$

For a reversible process, we have; $dS = \dfrac{\delta Q}{T}$; thus :

$$\delta W_{HE} = \delta Q - \left(-T_0 dS\right)$$

$$\delta Q = \delta W_{HE} - T_0 dS \quad (3)$$



Using Eqs. (1), (2), and (3); one can find:

$$\delta W_{total,useful} = \delta W_{HE} + \delta W_{b,useful} = -dU - P_0 dV + T_0 dS$$

Integrating from initial state to dead state gives:

$$W_{total,useful} = \left(U - U_0\right) + P_0\left(V - V_0\right) - T_0\left(S - S_0\right)$$

where $W_{total,useful}$ is the total useful work delivered as the system undergoes a reversible process from the given state to the dead state which is the exergy of the system. Total exergy for a closed system including the potential and kinetic energies can be written as:

$$X = \left(U - U_0\right) + P_0\left(V - V_0\right) - T_0\left(S - S_0\right) + m\frac{V^2}{2} + mgz$$

on a unit mass basis, the closed system exergy is :

$$\phi = \left(\mathrm{u} - \mathrm{u}_0\right) + P_0\left(v - v_0\right) - T_0\left(s - s_0\right) + \frac{V^2}{2} + gz$$

where subscript 0 denotes the state of the system at the dead state.

Exergy of a Flow Stream

A flowing fluid has flow energy; that is the energy needed to maintain flow in a pipe or line;

$$w_{flow} = Pv$$

The flow work is the boundary work done by a fluid on the fluid downstream. The exergy associated with flow energy can be written as

$$x_{flow} = PV - P_0V = (P - P_0)\ V$$

The exergy of a flow stream can be found from:

$$x_{flowing\ fluid} = x_{nonflowing\ fluid} + x_{\ flow}$$

$$x_{flowing\ fluid} = (u - u_0) + P_0(V - V_0) - T_0(s - s_0) + \mathbf{V}^2/2 + gz + (P - P_0)V$$

where $\mathbf{V}$ is the velocity of the system and V is the volume.

$$x_{flowing\ fluid} = (u + PV) - (\ u_0 - P_0\ V_0) - T_0(s - s_0) + \mathbf{V}^2/2 + gz$$

$$x_{flowing\ fluid} = \psi = (h - h_0) - T_0(s - s_0) + \mathbf{V}^2/2 + gz$$

The flow exergy is represented by symbol ψ.

Exergy by Heat Transfer

Heat is a form of disorganized energy, thus only a portion of it can be converted to work. Heat transfer Q at a location at thermodynamic temperature T is accompanied by exergy transfer:

$$X_{heat} = \left(1 - \frac{T_0}{T}\right)Q \qquad (kJ)$$

If the temperature of the heat source is changing with time, use:

$$X_{heat} = \int \left(1 - \frac{T_0}{T}\right)Q \qquad (kJ)$$

Exergy Transfer by Work

$$X_{work} = \begin{cases} W - W_{surr} & \text{for boundary work} \\ W & \text{for other forms of work} \end{cases}$$

where $W_{surr} = P_0\ (V_2 - V_1)$. Therefore the exergy transfer with work such as shaft work and electrical work is equal to the work W.

Note that the work done or against atmosphere is not available for any useful purpose, and should be excluded from available work.

The Decrease of Exergy Principle

Entropy and exergy for an isolated closed system can be related through an energy balance and entropy balance as follows:

1) Energy balance:

$E_{in} - E_{out} = \Delta E \rightarrow 0 = E_2 - E_1$ (1)

2) Entropy balance:

$S_{in} - S_{out} + S_{gen} = \Delta S_{system} \rightarrow S_{gen} = S_2 - S_1$ (2)

Multiplying Eq. (2) by T_0 and subtracting it from Eq. (1), one finds:

$$-T_0 S_{gen} = E_2 - E_1 - T_0 (S_2 - S_1)$$

For a closed system, we know that:

$$X_2 - X_1 = (E_2 - E_1) + P_0 \underbrace{(V_2 - V1)}_{=0} - T_0 (S_2 - S_1)$$

since $V_2 = V_1$ for an isolated system. Combining these two equations gives:

$$-T_0 S_{gen} = X_2 - X_1 \le 0$$

Since T0 is a positive value, we have:

$$\Delta X_{isolated} = (X_2 - X_1)_{isolated} \le 0$$

The exergy of an isolated system during a process always decreases or, in the limiting case of a reversible process, remains constant.

Irreversibilities such as friction, mixing, chemical reaction, heat transfer, unrestrained expansion always generate entropy (or destroy exergy).

$$X_{destroyed} = T_0 S_{gen} \ge 0$$

Exergy destruction cannot be negative. The decrease of exergy principle can also be expressed as:

$$X_{destroyed} = \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

Exergy Balance: Closed System

$X_{in} - X_{out} - X_{destroyed} = \Delta X_{system}$

where; $X_{destroyed} = T_0 S_{gen}$

For a closed system that does not involve any mass flow. The exergy balance can be written as:

$$\sum \left(1 - \frac{T_0}{T_k}\right) Q_k - [W - P_0 (V_2 - V_1)] - T_0 S_{gen} = X_2 - X_1$$

or in the rate form:

$$\sum \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k - \left[\dot{W} - P_0 \frac{dV_{system}}{dt}\right] - T_0 \dot{S}_{gen} = \frac{dX_{system}}{dt}$$

Exergy Balance: Control Volumes

Exergy balance relations for control volumes involve mass flow across the boundaries. The general exergy balance relationship can be expressed as:

$$X_{heat} - X_{work} + X_{mass,in} - X_{mass,out} - X_{destroyed} = (X_2 - X_1)_{CV}$$

Or,

$$\sum\left(1-\frac{T_0}{T_k}\right)Q_k - \left[W - P_0\left(V_2 - V_1\right)\right] + \sum_{in} m\psi - \sum_{out} m\psi - X_{destroyed} = \left(X_2 - X_1\right)_{CV}$$

in the rate form:

$$\sum\left(1-\frac{T_0}{T_k}\right)\dot{Q}_k - \left[\dot{W} - P_0\frac{dV_{CV}}{dt}\right] + \sum_{in}\dot{m}\psi - \sum_{out}\dot{m}\psi - \dot{X}_{destroyed} = \frac{dX_{CV}}{dt}$$

where the initial and final states of the control volume are specified , the exergy change of the control volume is:

$$X_2 - X_1 = m_2\phi_2 - m_1\phi_1$$

Steady-flow devices such as: turbines, compressors, nozzles, diffusers, heat exchangers, pipes, and ducts do not experience no changes in their mass, energy, entropy, and exergy content as well as their volumes. Therefore:

$$dV_{CV}/dt = 0$$

and

$$dX_{CV}/dt = 0$$

Thus, one can write:

$$\sum\left(1-\frac{T_0}{T_k}\right)\dot{Q}_k - \dot{W} + \sum_{in}\dot{m}\psi - \sum_{out}\dot{m}\psi - \dot{X}_{destroyed} = 0$$